1. Let $A$ be the following matrix,

$$
\begin{pmatrix}
0 & 1 & 0 \\
6 & 11 & 12 \\
-4 & -7 & -8
\end{pmatrix}
$$

(5 pts) (a): Find all eigenvalues of $A$.

(5 pts) (b): Is $A$ diagonalizable?
2. A reflection $A$ on $\mathbb{R}^n$ is defined to be a linear transformation $A$ of $\mathbb{R}^n$ which preserves the length of all vectors and reverse the orientation (i.e., with determinant $(A) = -1$). Let $A$ be a reflection on $\mathbb{R}^3$, show that $-1$ is an eigenvalue of $A$. 
3. Express the commutative group $\mathbb{Z}^3/(f_1, f_2, f_3)$ where $f_1 = (1, 2, 3)$, $f_2 = (4, 6, 8)$, $f_3 = (6, 10, 12)$ as a direct sum of cyclic groups.

4. A matrix $A$ is said to be \textit{idempotent} if $A^m = I$ for some $m \geq 1$. Show that a symmetric real matrix is idempotent iff $A^2 = 1$. 
(10 pts) 5. Let $<,>$ be an inner product of a finite dimensional complex vector space $V$, and $A$ a self-adjoint operator of $V$, Show that $< Av, v >$ is always a real number for any $v \in V$.

(10 pts) 6. Find the area of the convex pentagon in $\mathbb{R}^2$ with vertices $(0, 0), (6, 0), (8, 3), (5, 6), (0, 4)$. 

(10 pts)  7. Let \( P_3 \) be the vector space of all real polynomials of degree 3 or less. Let the inner product \((f|g)\) be defined as \( \int_0^1 fg \, dx \). Find an orthonormal basis of \( P_3 \).
8.
(10 pts) (a): Find the Jordan canonical form $J$ of the following matrix over complex numbers

$$A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(10 pts) (b): Find a matrix $M$ such that $J = M^{-1}AM$. 