NAME: ______________________________

T.T.Moh **Math 554 Qualifying Examination** **August 2012**

- This is a two hour test.
- Write your answers on the test paper!
- For decimal approximations, it is enough to give 2 decimal places.
- Show your work such that your reasoning can be followed.
- There are 10 pages, 10 questions, 20 points each and 200 points on this test.

1. Let $A \in M_{n\times n}(K)$ (the vector space of $n \times n$ matrices over a field K). State and prove the Cayley-Hamilton Theorem for A. :

2. Let $S = R[x]^3/(f_1, f_2, f_3)$ be a module over$R[x]$ where $R[x]$ is the ring of real polynomials and $f_1 = (x, 1, 0)$, $f_2 = (1, x, 0), f_3 = (0, 0, x-1)$ be three elements in $R[x]^3$. Express $R[x]^3/(f_1, f_2, f_3)$ as direct sum of modules $\oplus_{i=1}^{m} R[x]/(c_i)$ such that $m \leq 3$ and $c_i | c_{i+1}$.(The fundamental theorem of finitely generated modules over P.I.D.)

3. Show that the Z-module Z/nZ is not projective for integer $n \geq 2$.

4. Show that $Ext^1_Z(Z/mZ, Z) \approx Z/mZ$ for $m \geq 2$.

5. Find the best straight line fit (least square approximation) to the measurement $b = 1$ *at* $t = 0$, $b = 3$ *at* $t = 1$, $b = 3$ *at* $t = 2$.

6. Find an orthonormal basis for P_3, the vector space of all polynomials of degree ≤ 3 under the inner product defined as

$$< f|g >= \int_0^1 fg \; dx$$

7. Let R be the field of real numbers. Let W be the subspace of R^4 generated by $(1, 1, 0, 0)^T, (0, 0, 1, 1)^T$. Given $x = (1, 2, 3, 4)^T$. Find $y, z \in R^4$ such that $x = y + z$ and $y \in W, z \in W^{\perp}$.

8. Find an 2×2 matrix A which has all the principal minors positive and which is not a positive matrix.

9. Let A be the matrix over complex numbers as follows,

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 2 \\ 2 & 2 & 2 \end{pmatrix}.$$

Find the Jordan canonical form of A.

10. Decide if the following function has a local minima at the origin;

$$f(x, y, z) = 3x^2 + 6xy + 2xz + 4y^2 + 3yz + 2z^2$$