1. a) Define metric.
   b) Consider the set of all continuous real valued functions on \([0,1]\). Show 
   \[d(f,g) := \int_0^1 |f(x) - g(x)| \, dx\] defines a metric on \(X\).

2. Consider the topology on the real line generated by half open intervals \([x,y)\) and 
   \((x,y]\). Show that this is the discrete topology.

3. Show the unit interval \([0,1]\) is connected.

4. Let \(X\) be a compact Hausdorff space, let \(\{C_\alpha\}\) be a family of closed subsets so 
   that each \(C_\alpha \cap C_\beta\) is in the family. Let \(C = \cap C_\alpha\) and suppose that \(C \subset U\) where 
   \(U\) is an open set. Show \(C_\alpha \subset U\) for some \(\alpha\).

5. Give an example in the plane \(\mathbb{R}^2\) of a compact connected space which is not path 
   connected and not locally connected.

6. Let \(X\) be a Hausdorff space. Let the cone of \(X\), denoted \(CX\), be the quotient of 
   \(X \times [0,1]\) where \((x,1)\) is identified to a point. Show \(CX\) is locally compact only 
   if \(X\) is compact.

7. Let \(Y^X\) denote the space of continuous maps from \(X\) to \(Y\) given the compact-open 
   topology. Suppose \(X\) and \(Y\) are locally compact Hausdorff. Show that \(f\) and \(g\) 
   in \(Y^X\) are in the same path component if and only if they are homotopic (Hint: 
   Use the Exponential Law).

8. Suppose \(X\) is a path connected and locally path connected Hausdorff space so 
   that its universal covering space \(\tilde{X}\) exists. Assume \(X\) is compact. Show \(\tilde{X}\) is 
   compact only if \(\pi_1(X)\) is a finite group.

9. Give an example of two compact connected metric spaces which are homotopy 
   equivalent but are not homeomorphic.

10. Give an example of a connected finite simplicial complex \(K\) so that \(X(K) = -1\). 
    Recall that the Euler characteristic

\[
X(K) := \#(0 - \text{Simplexes}) - \#(1 - \text{Simplices}) + \#(2 - \text{Simplicies}) \ldots
\]