MA 571 Qualifying Exam. August 1996.

Each problem is worth 11 points.

1. Let $X$ be a set, let $B$ be a basis for a topology $T$ on $X$, and let $B'$ be a basis for another topology $T'$ on $X$. Give a condition involving $B$ and $B'$ which is equivalent to the condition that $T'$ is finer than $T$ (recall that this means that every $T$-open set is also $T'$-open). **Prove** that your answer is correct.

2. Let $A \subset X$ and $B \subset Y$. Show that in the space $X \times Y$,
   $$\overline{A \times B} = \overline{A} \times \overline{B}.$$

3. (a) Give an example of a space which is connected but not path-connected. You do not have to prove that your answer is correct.
   (b) Give a metric space in which not every closed and bounded subset is compact. You do not have to prove that your answer is correct.

4. Prove that every compact subset of a Hausdorff space is closed.

5. Show that if $Y$ is compact, then the projection map $X \times Y \to X$ is a closed map.

6. Prove that the one-point compactification of a locally-compact Hausdorff space is compact.

7. Let $I$ be the unit interval, and let $Y$ be a path-connected space. Prove that any two maps from $I$ to $Y$ are homotopic.

8. Let $p : E \to B$ be a covering map. Assume that $B$ is connected and locally connected. Show that if $C$ is a component of $E$, then $p|C : C \to B$ is a covering map.

9. Show that if $B$ is simply connected, then any covering map $p : E \to B$ for which $E$ is path connected is one-to-one.