MA 571 Qualifying Exam. August 1996.

Each problem is worth 11 points.

1. Let X be a set, let $\mathcal{B}$ be a basis for a topology $\mathcal{T}$ on X, and let $\mathcal{B}'$ be a basis for another topology $\mathcal{T}'$ on X. Give a condition involving $\mathcal{B}$ and $\mathcal{B}'$ which is equivalent to the condition that $\mathcal{T}'$ is finer than $\mathcal{T}$ (recall that this means that every $\mathcal{T}$-open set is also $\mathcal{T}'$-open). **Prove** that your answer is correct.

2. Let $A \subset X$ and $B \subset Y$. Show that in the space $X \times Y$,

$$\overline{A \times B} = \overline{A} \times \overline{B}.$$

3. (a) Give an example of a space which is connected but not path-connected. You do not have to prove that your answer is correct.

   (b) Give a metric space in which not every closed and bounded subset is compact. You do not have to prove that your answer is correct.

4. Prove that every compact subset of a Hausdorff space is closed.

5. Show that if Y is compact, then the projection map $X \times Y \to X$ is a closed map.

6. Prove that the one-point compactification of a locally-compact Hausdorff space is compact.

7. Let I be the unit interval, and let Y be a path-connected space. Prove that any two maps from I to Y are homotopic.

8. Let $p : E \to B$ be a covering map. Assume that B is connected and locally connected. Show that if C is a component of E, then $p|C : C \to B$ is a covering map.

9. Show that if B is simply connected, then any covering map $p : E \to B$ for which E is path connected is one-to-one.