1. An embedding is an injective continuous map which is a homeomorphism onto its image.
   a) Let \( f : X \to Y \) be a one to one continuous map, and let \( X \) be compact. Show that \( f \) is an embedding.
   b) Give an example of a map \( f : \mathbb{R}^1 \to \mathbb{R}^2 \) which is one to one but is not an embedding.

2. Let \( Y = \prod_{\alpha \in A} X_\alpha \) be the product of a family of spaces \( \{X_\alpha | \alpha \in A\} \) with the product topology. Show that \( Y \) is connected if and only if \( X_\alpha \) is connected for all \( \alpha \).
   a) When \( A \) is finite
   b) When \( A \) is arbitrary

3. Let \( X \) be a locally compact space.
   a) Show \( X \) is completely regular.
   b) Let \( A \) be a subspace homeomorphic to the unit interval \( I \). Show there exists a retraction \( r : X \to A \).

4. The Hahn-Mazurkiewicz theorem [Hocking and Young p.129]. Let \( X \) be a Hausdorff space. We say \( X \) is a Peano space if there exists a surjective map \( f : I \to X \). Then \( X \) is Peano space if and only if \( X \) is compact, connected, locally connected, and metrizable.
   a) Describe a map \( I \to I \times I \) which is onto.
   b) Show the product of arbitrarily many unit intervals may not be metrizable.
   c) Give an example of a closed, connected bounded set in \( \mathbb{R}^2 \) which is not the image of some \( f : I \to \mathbb{R}^2 \).
   d) Suppose that \( p : \tilde{X} \to X \) is the simply connecting covering space over a Peano space \( X \). Show that \( \tilde{X} \) is a Peano space if and only if \( \pi_1(X) \) is finite.
5. Let $f : X \to Y$ be a closed map onto a compact space $Y$ such that every fiber $f^{-1}(y)$ is compact. Show that $X$ is compact.