Qualifying Examination
MA 571
January 1997

Assume:	All spaces are Hausdorff. All maps are continuous.
	Products have the product topology.

1. An embedding is an injective continuous map which is a homeomorphism onto its image.

   a) Let $f : X \to Y$ be a one to one continuous map, and let X be compact. Show that f is an embedding.

   b) Give an example of a map $f : \mathbb{R}^1 \to \mathbb{R}^2$ which is one to one but is not an embedding.

2. Let $Y = \prod_{\alpha \in \mathcal{A}} X_\alpha$ be the product of a family of spaces $\{X_\alpha | \ \alpha \in \mathcal{A}\}$ with the product topology. Show that Y is connected if and only if X_α is connected for all α.

   a) When $\mathcal{A}$ is finite

   b) When $\mathcal{A}$ is arbitrary

3. Let X be a locally compact space.

   a) Show X is completely regular.

   b) Let A be a subspace homeomorphic to the unit interval I. Show there exists a retraction $r : X \to A$.

4. The Hahn-Mazurkiewicz theorem [Hocking and Young p.129]. Let X be a Hausdorff space. We say X is a Peano space if there exists a surjective map $f : I \to X$. Then X is Peano space if and only if X is compact, connected, locally connected, and metrizable.

   a) Describe a map $I \to I \times I$ which is onto.

   b) Show the product of arbitrarily many unit intervals may not be metrizable.

   c) Give an example of a closed, connected bounded set in $\mathbb{R}^2$ which is not the image of some $f : I \to \mathbb{R}^2$.

   d) Suppose that $p : \tilde{X} \to X$ is the simply connecting covering space over a Peano space X. Show that $\tilde{X}$ is a Peano space if and only if $\pi_1(X)$ is finite.

5. Let $f : X \to Y$ be a closed map onto a compact space Y such that every fiber $f^{-1}(y)$ is compact. Show that X is compact.