1. (10 pts) Let $X$ be a compact space and let

$$A_1 \supset A_2 \supset \cdots \supset A_k \cdots$$

be a descending chain of non-empty closed subsets of $X$. Show that the intersection $\bigcap_{k=1}^{\infty} A_k$ is not empty.

2. (10 pts) Show that topology of a compact metric space has a countable basis.

3. (10 pts) Show that the product of connected spaces is connected.

4. (10 pts) If $X$ is a simply connected space (i.e. every map $S^1 \to X$ is homotopic to a constant map) then every map $X \to S^1$ is homotopic to a constant map. Hint: Use the universal cover of $S^1$.

5. (20 pts) In each of the following describe a homeomorphism between the two topological spaces or show there is none.
   - $\mathbb{R}$ and $\mathbb{R}^2$
   - $\mathbb{R}^2$ and $D^2 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$
   - $[0, 1) \times (0, 1)$ and $[0, 1) \times [0, 1)$
   - The parabola $y = x^2$ with the subspace topology and $\mathbb{R}$

6. (10 pts) Let $M$ be a compact metric space, $X$ be a Hausdorff space and $f: M \to X$ be a continuous surjection. Show that $X$ is metrizable.