QUALIFYING EXAMINATION
AUGUST, 2002
MATH 571 - J. KAMINKER

Each part of a problem is worth 5 points, except that Problem 3 and Problem 6 are worth 10 points each.

1) Let X be a Hausdorff space.

a) Define: X is a normal space.
b) Prove that a metric space is normal.
c) Prove that any finite Hausdorff space is normal.

2) Let X and Y be path connected spaces.

a) Show that $X \times Y$ is path connected also.
b) Assume that X and Y are path connected subsets of $\mathbb{R}^2$. Show that $X \cup Y$ is path connected if $X \cap Y \neq \emptyset$. Is the converse true? Give a counterexample or prove it.

3) Prove that a compact subset of a Hausdorff space is closed.

4) Let X be a space and $A \subseteq X$ a subspace. Define X/A to be the quotient of the equivalence relation on X given by

$$x \sim y \Leftrightarrow x = y \quad \text{or} \quad x, y \in A$$

Let $\pi : X \to X/A$ be the quotient map.

a) Define the *quotient topology* on X/A.
b) Prove that $[0,1]/\{0,1\}$ is homeomorphic to the unit circle, S^1

5) Let X and Y be locally compact Hausdorff spaces and let $f : X \to Y$ be a continuous function. The map f is *proper* if $f^{-1}(K)$ is compact for every compact subset $K \subseteq Y$.

a) Define the *one-point compactification* of a locally compact Hausdorff space X.
b)Prove that if a map $f : X \to Y$ is proper then it extends to a continuous function $f^+ : X^+ \to Y^+$ between the one-point compactifications of X and Y.

6) Let $p : \tilde{X} \to X$ be a covering space. If $\tilde{x}_0 \in \tilde{X}$ and $p(\tilde{x}_0) = x_0$ show that the induced homomorphism, $p_* : \pi_1(\tilde{X}, \tilde{x}_0) \to \pi_1(X, x_0)$, is one-one.