QUALIFYING EXAMINATION
AUGUST 2003
MATH 571 - Prof. Smith

(10)I. The Intermediate Value theorem states that, for every continuous function $f\colon [a,c] \to \mathbf{R}$ from a closed interval to the real line, if $f(a) < r < f(b)$ then there exists $b \in [a,c]$ such that $f(b) = r$. Use properties of connected spaces to prove the Intermediate Value theorem.

(10)II. a) Prove that every open subspace of a compact Hausdorff topological space is locally compact.

b) Conversely prove that every locally compact Hausdorff space can be embedded as an open subspace of a compact Hausdorff space.

(5)III. Prove that the plane R^2 is not homeomorphic to three space R^3.

(5)IV. Which subspaces of a compact Hausdorff space are compact.

(10)V. Suppose $f\colon M \to X$ is a continuous function from a compact metric space M to a Hausdorff space X and that $f(M) = X$. State three topological properties of X (other than Hausdorff). Briefly state a reason that each property holds; a complete proof is not necessary.

(10)VI. Recall that a continuous function $S^1 \to B$ on the circle is null homotopic if it extends to a continuous function $S^1 \subseteq D^2 \to B$ on the disk. Let $\pi\colon E \to B$ be a covering space such that $\pi(E) = B$ and let X be a path connected topological space. Suppose that $f\colon X \to B$ is a continuous function and that for every continuous function $g\colon S^1 \to X$ the composition $f \circ g\colon S^1 \to B$ is null homotopic. Prove that there is a lift $l\colon X \to E$, i.e. that there is a continuous function l such that $\pi \circ l = f$. How many lifts are there?