QUALIFYING EXAMINATION
JANUARY 2003
MATH 571 - Prof. Gottlieb

1. Let $f : X \to Y$ be a continuous map. Given an open neighborhood V of a point $y \in Y$, we say that $f^{-1}(V)$ is a *tubular neighborhood* of the *fibre* $f^{-1}(y)$. We say f is *tubular* if for any open $U \subset X$ containing a fibre, there is a tubular neighborhood of that fibre contained in U. That is $f^{-1}(y) \subset f^{-1}(V) \subset U$.
   a) Prove that f is tubular if and only if f is a closed map.
   b) Show that f tubular implies that f is a quotient map.
   c) Give an example of a quotient map which is not tubular.
   d) Show that the Projection $\pi : B \times F \to B$ is a closed map if F is compact.

2. a) State the Tietze extension theorem.
   b) Let $f : X \to \mathbb{R}^3$ be a map where X is compact and connected. Let $\pi : \mathbb{R}^3 \to \mathbb{R}$ be the projection onto the x-axis and let $p : f(X) \to \pi(f(X))$ be the projection π restricted to $f(X)$ mapped onto its image. Then show p extends to a map $p' : \mathbb{R}^3 \to \pi(f(X))$.

3. Show that a metric space is separable if and only if it has a countable basis.

4. Prove that the unit disk D^2 does not retract onto its boundary circle S^1.

5. Show that the figure eight space, ∞, is homotopy equivalent to the theta space, θ. (Hint: One way to show this, they are both homotopy equivalent to a third space.)

6. State the Unique Path Lifting Property for covering spaces and explain how it leads to the Unique Homotopy Lifting Property for covering spaces.