QUALIFYING EXAMINATION
JANUARY 2004
MATH 571 - Prof. Smith

1. A space is second countable if it has a countable basis. Let $X = \mathbb{R}^{\mathbb{N}}$ be the product of a countable number of copies of $\mathbb{R}$. Show that the product topology on X is second countable. Give an example of a topology of X which is not second countable.

2. Let X be a non-empty compact connected space and let $f : X \to \mathbb{R}$ be a continuous function to the real line. Describe the subspace $f(X)$ as completely as possible. Which two theorems of calculus does this generalize?

3. Show that every open subspace of a locally compact Hausdorff space is Hausdorff. (Hint: compactify).

4. Let X be a compact Hausdorff space and let $A_1 \supset A_2 \supset \cdots$ be a descending chain of closed connected subspaces of X. Prove that

$$A = \bigcap_{i=1}^{\infty} A_i \quad \text{is connected.}$$

5. Prove that the map $p : \mathbb{C} - 0 \mapsto \mathbb{C} - 0$ from the punctured complex plane to itself given by $p(z) = z^n$ is a covering map for $n \neq 0$.

6. Let $\pi : E \to B$, be a covering map. Suppose that E and B are path connected and that B is simply connected. Prove that π is a homeomorphism.