QUALIFYING EXAMINATION
JANUARY 2006
MATH 571 - Prof. Smith

(20) I. a) Give an example of a subspace of R^2 that is connected but not path connected. Explain. b) Show that an open subspace of R^2 is connected if and only if it is path connected.

(20) II.For each integer $n > 0$ let C_n be the circle of radius n and center $(n, 0)$ and let D_n be the circle of radius $1/n$ and center $(1/n, 0)$. Each of these circles is tangent to the y-axis at the origin. Let $X = \cup_n C_n$ and $Y = \cup_n D_n$ have the subspace topology. a) Show that the subspace $X - \{(0,0)\}$ is homeomorphic to the subspace $Y - \{(0,0)\}$. b) But, X is not homeomorphic to Y. State three topological properties that distinguish them. Briefly explain which has the property and which does not.

(15) III. The two sphere $S^2 = \{(x, y, z) \,|\, x^2 + y^2 + z^2 = 1\}$ is a subspace of R^3. Let X be the quotient of S^2 obtained by identifying each point (x, y, z) of S^2 with $(-x, -y, -z)$. Show that the quotient map $S^2 \to X$ is a covering map.

(15) IV. Two of the fundamental theorems in calculus are special cases of more general results from point set topology. State the more general results and explain how they imply the intermediate value theorem and the theorem that every real-valued continuous function on an interval has a maximum value.

(15) V. Explain how to use the computation of $\pi_1 S^1$ to prove that every self map of the disk has a fixed point.

(15) VI. Let $f : E \to B$ be a covering map and assume that both E and B are path connected. Assume that $\pi_1 B$ is the cyclic group of order 10 and that $f^{-1}(b)$ has two elements. What is $\pi_1 E$?