MA 571 Qualifying Exam. August 2011. Professor McClure.

Each problem is worth 14 points and you get two points for free.

Unless otherwise stated, you may use anything in Munkres's book—but be careful to make it clear what fact you are using.

When you use a set theoretic fact that isn't stated in Munkres and isn't obvious, be careful to give a clear explanation.

1. Let X and Y be topological spaces, and let $p : X \times Y \to X$ be the projection. **Prove** that p is an open map (that is, it takes open sets of the product topology to open sets of X).

2. Let K denote the set $\{1/n \mid n \in \mathbb{Z}_+\}$ (where $\mathbb{Z}_+$ denotes the set of positive integers). What are the components of $(K \times [0, 1]) \cup \{(0, 0)\}$? **Prove** that your answer is correct.

3. Let X be a compact space and let $\{C_\alpha\}_{\alpha \in A}$ be a collection of closed sets in X. Let $C = \cap_{\alpha\in A} C_\alpha$ and let U be an open set containing C. **Prove** that there is a finite set $\alpha_1, \cdots, \alpha_n$ in A with $C_{\alpha_1} \cap \cdots \cap C_{\alpha_n} \subset U$.

4. Let X be a topological space and let A be a subset of X, with the subspace topology. Let $\sim$ be an equivalence relation on X. Then $\sim$ restricts to an equivalence relation on A and the map of sets $A/{\sim} \to X/{\sim}$ is 1–1 (you do **not** have to prove this).

   Let $\mathcal{T}$ be the quotient topology on $A/{\sim}$ (considered as a quotient of A) and let $\mathcal{T}'$ be the subspace topology on $A/{\sim}$ (considered as a subset of $X/{\sim}$).

   Prove that every set that's open for the topology $\mathcal{T}'$ is also open for the topology $\mathcal{T}$. As part of your proof, **explain** exactly how you are using properties of the subspace and quotient topologies.

5. Let S^1 be the unit circle in the complex plane.

   Suppose that X is a wedge of two circles: that is, X a Hausdorff space which is a union of two subspaces A_1, A_2 such that A_1 and A_2 are each homeomorphic to S^1 and $A_1 \cap A_2$ is a single point p.

   Suppose that Y is also a wedge of two circles: that is, Y a Hausdorff space which is a union of two subspaces B_1, B_2 such that B_1 and B_2 are each homeomorphic to S^1 and $B_1 \cap B_2$ is a single point q.

   Prove that X is homeomorphic to Y.

6. Let X and Y be topological spaces and let $x_0 \in X$, $y_0 \in Y$. **Prove** that there is a function from $\pi_1(X \times Y, (x_0, y_0))$ to $\pi_1(X, x_0) \times \pi_1(Y, y_0)$ which is 1–1 and onto (you do **NOT** have to show that it is a homomorphism).

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7. Let X be the quotient space obtained from a square P by pasting its edges together according to the labelling scheme $aba^{-1}b$.

   i) Calculate $H_1(X)$. (You may use anything proved in Munkres, but be sure to be clear about what you're using.)

   ii) Assuming X is homeomorphic to one of the standard surfaces in the classification theorem, which surface is it?