1. Let $X$ be a topological space and suppose that there is a countable collection of open sets
\[ \mathcal{B} = \{U_1, U_2, \ldots\} \]
which is a basis for the topology of $X$. Let $A \subset X$ and let $x \in \overline{A}$. \textbf{Prove} that there is a sequence in $A$ which converges to $x$.

2. Let $X$ be a topological space and let $A$ be a set with the property that each point of $A$ has a neighborhood $U$ with $A \cap U$ closed as a subspace of $U$. \textbf{Prove} that there is an open $V$ with $A = \overline{A} \cap V$.

3. \textbf{Prove} that there is an equivalence relation $\sim$ on the interval $[0, 1]$ such that $[0, 1]/\sim$ is homeomorphic to $[0, 1] \times [0, 1]$. As part of your proof explain how you are using one or more properties of the quotient topology.

4. Let $X$ be a Hausdorff space. Let $x$ be a point of $X$ and let $C$ be a compact set with $x \notin C$. \textbf{Prove} that there are disjoint open sets $U$ and $V$ with $x \in U$ and $C \subset V$.

5. (For this problem, you may use the fact stated in Problem 4, even if you didn’t do that problem). Let $Y$ be a locally compact Hausdorff space, let $y$ be a point of $Y$, and let $W$ be a neighborhood of $y$. \textbf{Prove} that there is a neighborhood $O$ of $y$ such that $\overline{O}$ is compact and $\overline{O} \subset W$.

6. Let $X$ be the quotient space obtained from an octagon $P$ by pasting its edges together according to the labelling scheme $abab^{-1}cdcd^{-1}$ (read the formula carefully!).
   i) Calculate $H_1(X)$. (You may use anything proved in Munkres, but be sure to be clear about what you’re using.)
   ii) Assuming $X$ is homeomorphic to one of the standard surfaces in the classification theorem, which surface is it?

7. Let $p : E \to B$ be a covering map. Let $A$ be a connected space and let $a \in A$. \textbf{Prove} that if two continuous functions $\alpha, \beta : A \to E$ have the property that $\alpha(a) = \beta(a)$ and $p \circ \alpha = p \circ \beta$ then $\alpha = \beta$.

\textbf{For partial credit} you may assume that $p$ is the standard covering map from $\mathbb{R}$ to $S^1$. 