MA 571 Qualifying Exam. August 2012. Professor McClure.

Each problem is worth 14 points and you get two points for free.

Please be careful that your handwriting is clear and easy to read.

Unless otherwise stated, you may use anything in Munkres's book—but be careful to make it clear what fact you are using.

When you use a set theoretic fact that isn't stated in Munkres and isn't obvious, be careful to give a clear explanation.

1. Let X be a topological space and suppose that there is a countable collection of open sets
$$\mathcal{B} = \{U_1, U_2, \ldots\}$$
which is a basis for the topology of X. Let $A \subset X$ and let $x \in \bar{A}$. **Prove** that there is a sequence in A which converges to x.

2. Let X be a topological space and let A be a set with the property that each point of A has a neighborhood U with $A \cap U$ closed as a subspace of U. **Prove** that there is an open V with $A = \bar{A} \cap V$.

3. **Prove** that there is an equivalence relation $\sim$ on the interval $[0,1]$ such that $[0,1]/\sim$ is homeomorphic to $[0,1] \times [0,1]$. As part of your proof **explain** how you are using one or more properties of the quotient topology.

4. Let X be a Hausdorff space. Let x be a point of X and let C be a compact set with $x \notin C$. **Prove** that there is are disjoint open sets U and V with $x \in U$ and $C \subset V$.

5. (For this problem, you may use the fact stated in Problem 4, even if you didn't do that problem). Let Y be a locally compact Hausdorff space, let y be a point of Y, and let W be a neighborhood of y. **Prove** that there is a neighborhood O of y such that $\bar{O}$ is compact and $\bar{O} \subset W$.

6. Let X be the quotient space obtained from an octagon P by pasting its edges together according to the labelling scheme $abab^{-1}cdc^{-1}d^{-1}$ (read the formula carefully!).

   i) Calculate $H_1(X)$. (You may use anything proved in Munkres, but be sure to be clear about what you're using.)

   ii) Assuming X is homeomorphic to one of the standard surfaces in the classification theorem, which surface is it?

7. Let $p : E \to B$ be a covering map. Let A be a connected space and let $a \in A$. **Prove** that if two continuous functions $\alpha, \beta : A \to E$ have the property that $\alpha(a) = \beta(a)$ and $p \circ \alpha = p \circ \beta$ then $\alpha = \beta$.

   For partial credit you may assume that p is the standard covering map from $\mathbb{R}$ to S^1.