# MA 571 Qualifying Exam. January 2012. Professor McClure.

Each problem is worth 14 points and you get two points for free.

**Unless otherwise stated, you may use anything in Munkres's book—but be careful to make it clear what fact you are using.**

**When you use a set theoretic fact that isn't stated in Munkres and isn't obvious, be careful to give a clear explanation.**

1. Let $X$ be a topological space. Recall that a subset of $X$ is *dense* if its closure is $X$. **Prove** that the intersection of two dense open sets is dense.

2. Let $X$ be a set with two elements $\{a, b\}$. Give $X$ the *indiscrete* topology. Give $X \times \mathbb{R}$ the product topology. Let $A \subset X \times \mathbb{R}$ be $(\{a\} \times [0, 1]) \cup (\{b\} \times (0, 1))$. **Prove** that $A$ is compact.

   You may use that fact that a set is compact if every covering by *basic* open sets has a finite subcovering.

3. Let $B^2$ be the disk $\{(x, y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 1\}$. Let $S^1$ be the circle $\{(x, y) \in \mathbb{R}^2 \mid x^2+y^2 = 1\}$. **Prove** that there is an equivalence relation $\sim$ such that $B^2$ is homeomorphic to $(S^1 \times [0, 1])/\sim$. As part of your proof **explain** how you are using one or more properties of the quotient topology.

4. Let $X$ be a set with 2 elements $\{a, b\}$. Give $X$ the *discrete* topology. Let $Y$ be any topological space. Recall that $\mathcal{C}(X, Y)$ denotes the set of continuous functions from $X$ to $Y$, with the compact-open topology. **Prove** that $\mathcal{C}(X, Y)$ is homeomorphic to $Y \times Y$ (with the product topology).

5. Let $X$ and $Y$ be homotopy-equivalent topological spaces. Suppose that $X$ is path-connected. **Prove** that $Y$ is path-connected.

6. Suppose that $X$ is a wedge of two circles: that is, $X$ a Hausdorff space which is a union of two subspaces $A_1$, $A_2$ such that $A_1$ and $A_2$ are each homeomorphic to $S^1$ and $A_1 \cap A_2$ is a single point $p$.

   Use the Seifert-van Kampen theorem to **calculate** $\pi_1(X, p)$. You should state what deformation retractions you are using, but you don't have to give formulas for them.

7. Let $p : E \to B$ be a covering map. Let $A$ be a connected space and let $a \in A$. **Prove** that if two continuous functions $\alpha, \beta : A \to E$ have the property that $\alpha(a) = \beta(a)$ and $p \circ \alpha = p \circ \beta$ then $\alpha = \beta$.

   **For partial credit** you may assume that $p$ is the standard covering map from $\mathbb{R}$ to $S^1$.