Galois representations associated to regular algebraic cuspidal automorphic representations of $GL_n$ are de Rham

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Abstract

I will discuss local-global compatibility at $p$ for the $p$-adic Galois representations constructed by Harris-Lan-Taylor-Thorne and Scholze. More precisely, let $r_p(\pi)$ denote an $n$-dimensional $p$-adic representation of the absolute Galois group of a CM field $F$ attached to a regular algebraic cuspidal automorphic representation $\pi$ of $GL_n(\mathbb{A}_F)$. For any prime $v | p$ of $F$, we show that $r_p(\pi)|_{\text{Gal} (\mathbb{F}_v/F_v)}$ is de Rham.

To prove the above, we use the fact that the representations $r_p(\pi)$ can be constructed as a subrepresentation of a $p$-adic Galois representation $R_p(\Pi)$ associated to an overconvergent $GU(n, n)$-automorphic representation $\Pi$. We can then construct a certain “twisted” one-parameter family containing $\Pi$ and a Zariski-dense set of points whose associated Galois representations are already known to be de Rham. Using results of R. Liu and S. Shah, we can show that each specialization within this family has $n$ periods, and conclude the result by proving that the resulting $n$ periods of $R_p(\Pi)$ are all distinct periods of $r_p(\pi)$. 