### MA 15400 Spring 2013 Exam 2

<table>
<thead>
<tr>
<th>Trigonometric Identity</th>
<th>[ u, v ]</th>
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<tbody>
<tr>
<td>[ \sin(u + v) = \sin u \cos v + \cos u \sin v ]</td>
<td>[ \sin(u - v) = \sin u \cos v - \cos u \sin v ]</td>
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<td>[ \tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} ]</td>
<td>[ \tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v} ]</td>
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<tr>
<td>[ \sin(2u) = 2 \sin u \cos u ]</td>
<td>[ \cos(2u) = \cos^2 u - \sin^2 u ]</td>
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<td>[ \tan(2u) = \frac{2 \tan u}{1 - \tan^2 u} ]</td>
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<tr>
<td>[ \sin^2 \theta + \cos^2 \theta = 1 ]</td>
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<tr>
<td>[ 1 + \tan^2 \theta = \sec^2 \theta ]</td>
<td>[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} ]</td>
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<tr>
<td>[ 1 + \cot^2 \theta = \csc^2 \theta ]</td>
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1. Express as a trigonometric function of one angle.

\[ \cos(49^\circ)\cos(24^\circ) - \sin(49^\circ)\sin(24^\circ) \]

A. \( \cos(73^\circ) \)
B. \( \sin(25^\circ) \)
C. \( \sin(73^\circ) \)
D. \( \cos(25^\circ) \)
E. None of the above

2. Find all solutions of the equation using \( n \) as an arbitrary integer.

\[ \sin\left(2x - \frac{\pi}{3}\right) = \frac{1}{2} \]

A. \( x = \frac{\pi}{3} + \pi n, \frac{\pi}{2} + \pi n \)
B. \( x = \frac{\pi}{6} + \pi n, \frac{\pi}{2} + \pi n \)
C. \( x = \frac{\pi}{4} + \pi n, \frac{5\pi}{12} + \pi n \)
D. \( x = \frac{\pi}{4} + \pi n, \frac{7\pi}{12} + \pi n \)
E. None of the above

3. Find the solutions of the equation that are in the interval \([0, 2\pi)\).

\[ 2\sin^2 u = -1 + 3\sin u \]

A. \( \frac{\pi}{3}, \frac{2\pi}{3}, \frac{\pi}{2} \)
B. \( \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2} \)
C. \( \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{3\pi}{2} \)
D. \( \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2} \)
E. None of the above
4 and 5: A ship leaves port at 1:00 pm and sails in the direction N40°W at a rate of 22 miles per hour. A second ship leaves the same port at 2:00 pm and sails in the direction N50°E at a rate of 15 miles per hour.

4. To the nearest mile, approximately how far apart are the ships at 5:00 pm?
   A. 80 miles
   B. 95 miles
   C. 99 miles
   D. 107 miles
   E. None of the above

5. To the nearest degree, what is the bearing from the first ship to the second at 5:00 pm?
   A. S67°E
   B. S59°E
   C. S27°E
   D. S13°E
   E. None of the above
6. Determine if the identity is correct.

\[ \cos \left( \theta + \frac{\pi}{3} \right) = \frac{1}{2} (\sqrt{3} \cos \theta - \sin \theta) \]

A. Yes, it is correct.
B. No, it is not correct.

7. Find the solutions of the equation that are in the interval \([0, 2\pi)\).

\[ \cos \left( 3x - \frac{\pi}{6} \right) = -1 \]

A. \( \frac{4\pi}{9}, \frac{10\pi}{9}, \frac{16\pi}{9} \)
B. \( 0, \frac{2\pi}{3}, \frac{4\pi}{3} \)
C. \( \frac{7\pi}{18}, \frac{19\pi}{18}, \frac{31\pi}{18} \)
D. \( \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8} \)
E. None of the above

8. If \( \sin \alpha = -\frac{3}{8} \) and \( \cos \beta = \frac{4}{5} \) for a third-quadrant angle \( \alpha \) and a first-quadrant angle \( \beta \), then find the exact value of \( \sin(\alpha - \beta) \).

A. \( \frac{-12 + 3\sqrt{55}}{40} \)
B. \( \frac{9 - 4\sqrt{55}}{40} \)
C. \( \frac{12 - 3\sqrt{55}}{40} \)
D. \( \frac{-9 + 4\sqrt{55}}{40} \)
E. None of the above
9. A ladder 33.0 feet long leans against the side of a building, and the angle between the ladder and the building is $25^\circ$. If the distance from the bottom of the ladder to the building is increased by 4.0 feet, approximately how far does the top of the ladder move down the building? Give the answer to one decimal place.

A. 1.8 feet  
B. 2.2 feet  
C. 1.9 feet  
D. 2.0 feet  
E. None of the above

10. Find the exact value of $\tan(2\theta)$ for the given value of $\theta$.

$\tan \theta = \frac{-3}{2}, \ 90^\circ < \theta < 180^\circ$

A. $\tan (2\theta) = \frac{20}{21}$  
B. $\tan (2\theta) = \frac{-12}{5}$  
C. $\tan (2\theta) = \frac{-20}{21}$  
D. $\tan (2\theta) = \frac{12}{5}$  
E. None of the above
11. Find the solutions of the equation that are in the interval \([0, 2\pi)\).

\[
\sin t + \sin(2t) = 0
\]

A. \( t = \frac{\pi}{2}, \frac{3\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{3} \)

B. \( t = 0, \frac{2\pi}{3}, \frac{4\pi}{3} \)

C. \( t = \frac{\pi}{2}, \frac{3\pi}{4}, \frac{2\pi}{3}, \frac{4\pi}{3} \)

D. \( t = 0, \frac{\pi}{3}, \frac{5\pi}{3} \)

E. None of the above

12. If a projectile is fired from ground level with an initial velocity of \(v\) ft/sec and at an angle of \(\theta\) degrees with the horizontal, the range \(R\) of the projectile is given by the following formula. If \(v = 87\) ft/sec, approximate the angles, to the nearest whole degree, that result in a range of 151 feet.

\[
R = \frac{v^2}{16} \sin \theta \cos \theta
\]

A. \( \theta = 40^\circ, 50^\circ \)

B. \( \theta = 21^\circ, 69^\circ \)

C. \( \theta = 42^\circ, 48^\circ \)

D. \( \theta = 20^\circ, 70^\circ \)

E. None of the above
13. Find the exact value of the expression whenever it is defined.

\[ \cos\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) \]

A. \(-\frac{\sqrt{3}}{2}\)

B. \(\frac{1}{2}\)

C. \(\frac{\sqrt{3}}{2}\)

D. \(-\frac{1}{2}\)

E. None of the above

14. Write the expression as an algebraic expression in \(x\) for \(x > 0\).

\[ \cos\left(\tan^{-1} x\right) \]

A. \(\frac{x}{\sqrt{1+x^2}}\)

B. \(\frac{1}{1+x}\)

C. \(\frac{x}{1+x}\)

D. \(\frac{1}{\sqrt{1+x^2}}\)

E. None of the above

15. Approximate the solutions of the equation, to two decimals, that are in the given interval.

\[ \sin^2 x - 2\sin x - 2 = 0; \quad \left[0, 2\pi\right) \]

A. 3.96, 5.46

B. 2.36, 3.18

C. 2.73, 3.87

D. 3.57, 5.14

E. None of the above
Exam 2 Answers

1. $\cos(73^\circ)$ A
2. $x = \frac{\pi}{4} + \pi n, \frac{7\pi}{12} + \pi n$ D
3. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$ B
4. 99 miles C
5. S67°E A
6. No, it is not correct. B
7. $\frac{7\pi}{18}, \frac{19\pi}{18}, \frac{31\pi}{18}$ C
8. $\frac{-12 + 3\sqrt{55}}{40}$ A
9. 2.2 feet B
10. $\tan(2\theta) = \frac{12}{5}$ D
11. $t = 0, \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$ B
12. $\theta = 20^\circ, 70^\circ$ D
13. $\frac{\sqrt{3}}{2}$ C
14. $\frac{1}{\sqrt{1 + x^2}}$ D
15. 3.96, 5.46 A