MA161 Final Exam, Fall 2013, December 9, 7:00-9:00pm

Name:______________________________________________________________

10-digit PUID Number:________________________________________________

Recitation Instructor:_________________________________________________

Recitation Section Number and Time:____________________________________

Mark TEST 01 on your scantron!

Instructions:

1. Fill in all the information requested above and on the scantron sheet. On the scantron sheet fill in the little circles for your name, section number, and PUID.
2. This booklet contains 22 problems. The test booklet has five pages including this one.
3. For each problem mark your answer on the scantron sheet and also circle it in this booklet.
4. Work only on the pages of this booklet.
5. No books, notes, calculators, or any kind of electronic device are to be used during this test.
6. At the end of the exam turn in your exam and scantron sheet to your recitation instructor.
1. Find the constants \( a \) and \( b \) such that

\[
F(x) = \begin{cases} 
\frac{x^2 - x}{x - 1}, & x < 1 \\
ax^2 + bx + 1, & 1 \leq x < 2 \\
x + a, & x \geq 2
\end{cases}
\]

is continuous for all \( x \).

A. \( a = 3, \ b = -1 \)
B. \( a = 1, \ b = 1 \)
C. \( a = -1, \ b = 3 \)
D. \( a = 1, \ b = -1 \)
E. \( a = 2, \ b = 0 \)

2. A sample of a radioactive element initially has mass of 24 gm. After 2 minutes the sample of that element has mass of 2 gm. When (in minutes) is the mass equal to 4 gm?

A. \( \frac{2 \ln 6}{\ln 12} \)
B. \( \frac{\ln 6}{\ln 12} \)
C. \( \frac{2 \ln 12}{\ln 6} \)
D. \( \frac{3 \ln 6}{2 \ln 2} \)
E. \( \frac{3 \ln 8}{\ln 3} \)
3. Let \( h(x) = f(g(x)) \), where \( g(1) = 2 \), \( g'(1) = 3 \), \( f(1) = 4 \), \( f'(1) = 5 \), \( f(2) = 6 \), and \( f'(2) = 7 \). Then \( h'(1) = \)

A. 5  
B. 7  
C. 15  
D. 21  
E. None of the above is correct  

4. A rectangular cardboard box with no top has a rectangular base so that one side is twice as long as the other. If the box must have a volume of \( \frac{4}{3} \) m³, what should the height of the box be to minimize the amount of cardboard used?

A. \( \left( \frac{2}{3} \right)^{\frac{1}{3}} \) m  
B. \( \frac{2}{3} \) m  
C. \( \frac{1}{\sqrt{2}} \) m  
D. \( \frac{2\sqrt{2}}{3} \) m  
E. \( \frac{2}{\sqrt{3}} \) m
5. What is the domain of \( f(x) = \sqrt{4x - x^2} + \ln(1 - x) \)?
   A. \([0, 1) \cup (1, 4]\)
   B. \([0, 1]\)
   C. \((1, 4]\)
   D. \([1, 4]\)
   E. \((0, 1]\)

6. Evaluate \( \lim_{x \to 0^+} \left( \frac{1}{x} - \frac{1}{e^x - 1} \right) \)
   A. \(\infty\)
   B. 0
   C. \(\frac{1}{2}\)
   D. 2
   E. \(-\infty\)
7. The slope of the line tangent to the graph of

\[ y^2 - 2x^3 + xy^4 + 13 = 0 \]

at \((2,1)\) is

A. \(\frac{12}{5}\)  
B. \(\frac{18}{5}\)  
C. \(-\frac{5}{18}\)  
D. \(\frac{23}{10}\)  
E. \(-13\)

8. \(\int x^3(1 - 2x^4)^{1/4} \, dx = \)

A. \(-\frac{5}{32}(1 - 2x^4)^{5/4} + C\)  
B. \(-\frac{1}{8}(1 - 2x^4)^{5/4} + C\)  
C. \(\frac{4}{5}(1 - 2x^4)^{5/4} + C\)  
D. \(\frac{2}{5}(1 - 2x^4)^{5/4} + C\)  
E. \(-\frac{1}{10}(1 - 2x^4)^{5/4} + C\)
9. Suppose that \( \lim_{x \to 2} f(x) = 0 \) and \( \lim_{x \to 2} g(x) = \infty \).

What can be said about \( \lim_{x \to 2} f(x)g(x) \)?

A. \( \lim_{x \to 2} f(x)g(x) = 0 \)
B. \( \lim_{x \to 2} f(x)g(x) = \infty \)
C. \( \lim_{x \to 2} f(x)g(x) = 1 \)
D. \( \lim_{x \to 2} f(x)g(x) \) is either 0 or \( \infty \)
E. The limit may or may not exist. If it exists it can be any number.

10. The graph of the function \( f(x) = (1/2)^x \) looks most like:
11. \( \lim_{t \to 0} \sin(1/t^2) = \)
   
   A. 1
   B. \( \frac{1}{2} \)
   C. 0
   D. \( \infty \)
   E. The limit does not exist.

12. Use linear approximation to estimate the value of \( \cosh(1.1) \) rounded to 2 decimal places. Note that \( \cosh(1) \approx 1.54 \) and \( \sinh(1) \approx 1.18 \).
   
   A. 1.42
   B. 1.66
   C. 1.30
   D. 1.06
   E. 1.58
13. Find the number $c$ that satisfies the conclusion of the Mean Value Theorem on $[0, 9]$ if $f(x) = 2\sqrt{x}$.

A. $c = \frac{9}{4}$
B. $c = 0$
C. $c = \frac{1}{4}$
D. $c = 5$
E. No such number exists in $(0, 9)$

14. Compute the limit

$$
\lim_{x \to 0} \frac{e^{2x} - 1}{\tan x}
$$

A. $-1$
B. 0
C. 1
D. 2
E. $-2$
15. According to the Intermediate Value Theorem, the function \( f(x) = e^x + 3x - 4 \) has a zero in the interval
   A. \((-1, 0)\)
   B. \((0, 1)\)
   C. \((2, 4)\)
   D. \((1, 2)\)
   E. Both A. and B.

16. For which real number \( a \) does
   \[
   \lim_{r \to 5} \frac{ar^2 + 25}{r - 5}, \quad \text{exist?}
   \]
   A. \( a = -2 \)
   B. \( a = -1 \)
   C. \( a = 1 \)
   D. \( a = 2 \)
   E. \( a = -5 \)
17. If \( f(x) = \frac{x^2e^{2x}}{(x^2 - 1)} \), then \( f'(x) = \)

A. \( \frac{x^3e^{2x}(x^2 - x + 1)}{(x^2 - 1)^2} \)

B. \( \frac{2xe^{2x}(x^3 - x - 1)}{(x^2 - 1)^2} \)

C. \( \frac{2x^3e^{2x}(x^3 - x - 1)}{(x^2 - 1)^2} \)

D. \( \frac{2xe^{2x}(x^3 - x - 1)}{(x^2 - 1)^2} \)

E. \( \frac{2xe^{2x}(x^3 - x - 1)}{(x^2 - 1)} \)

18. If \( f(x) = x^{\sqrt{3}} \), then \( f'(4) = \)

A. \( 4 \ln 4 + 4 \)

B. \( 16 \ln 4 + 16 \)

C. \( 4 \ln 4 + 8 \)

D. \( 16 \ln 4 + 8 \)

E. \( 4 \ln 4 + 16 \)
19. Sand is dumped at a rate of 24 ft$^3$/min onto a pile whose shape is a cone whose base diameter and height are always equal. How fast is the height of the pile increasing (in ft/min) when the pile is 8 ft high? ($V = \frac{1}{3} \pi r^2 h$)

A. $\frac{8}{3\pi}$

B. $\frac{3}{2\pi}$

C. $\frac{2}{\pi}$

D. $\frac{3}{4\pi}$

E. $\frac{4}{3\pi}$

20. A cylindrical soup can is to have a volume of 1 cubic foot. The material for the side of the can costs $5/ft^2$, while the material for the top and bottom costs $2/ft^2$. What is the radius of a can that minimizes the cost of manufacturing? (The volume of the can is $\pi r^2 h$, the area of the top is $\pi r^2$, and the area of the side is $2\pi rh$.)

A. $\left(\frac{5}{4\pi}\right)^{1/3}$

B. $\left(\frac{5}{8\pi}\right)^{1/3}$

C. $2\sqrt{5\pi} + 20$

D. $10\pi + 20$

E. $1/5$
21. If

\[ f(x) = \int_{a}^{\sin^2(x)} g(t) dt \]

then \( f'(x) = \)

A. \( g(\sin^2(x))g'(x) \)
B. \( g(\sin^2(x)) \)
C. \( g(\sin^2(x))2 \sin(x) \cos(x) \)
D. \( g'(\sin^2(x)) \)
E. \( g'(\sin^2(x))2 \sin(x) \cos(x) \)

22. Calculate

\[ \int_{0}^{\pi/4} \frac{1}{\cos^2(x) \sqrt{1 + 2 \tan(x)}} dx \]

A. 0
B. \( \sqrt{3} - 1 \)
C. 1
D. \( \sqrt{3} \)
E. 2