MA 162 Exam 2 Fall 2002

NAME ____________________

STUDENT ID ____________________

REC. INSTR. ____________ REC. TIME. ____________

INSTRUCTOR ____________________

INSTRUCTIONS:

1. Make sure that you have all 6 test pages.
2. Fill in your name, your student ID number, and your instructor's name above.
3. There are 12 problems.
4. No books or notes or calculators may be used.

Midpoint Rule

$$M_n = \frac{b-a}{n}[f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)] \text{ where } \bar{x}_i = \frac{1}{2}(x_{i-1} + x_i).$$

Trapezoidal Rule

$$T_n = \frac{b-a}{2n}[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)].$$

Simpson's Rule

$S_n = \frac{b-a}{3n}[f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$ where n is even.

Let R be the region between the graphs of f and g on $[a, b]$. Then the moments of R about x and y axes are

$$M_x = \int_a^b \frac{1}{2}(f(x)^2 - g(x)^2)dx$$

$$M_y = \int_a^b x(f(x) - g(x))dx.$$

(10 pts) 1. Evaluate $\int \cos^5 x \sin^4 x\, dx$.

(10 pts) 2. $\displaystyle\int_0^2 \frac{x}{\sqrt{x^2+4}}\, dx =$

(A) $\frac{8}{3}(2\sqrt{2}-1)$

(B) $2(\sqrt{2}-1)$

(C) $2\sqrt{2}$

(D) $\frac{8}{3}\sqrt{2}$

(E) $4(\sqrt{2}-1)$

(12 pts) 3. $\displaystyle\int_1^2 \frac{x+3}{x^2+3x+2}\,dx =$

(A) $\ln\left(\frac{1}{27}\right)$

(B) $\ln\left(\frac{16}{27}\right)$

(C) $\ln\left(\frac{27}{16}\right)$

(D) $\ln 9$

(E) $\ln 27$

(10 pts) 4. $\displaystyle\int_3^4 x\sqrt{3-x}\,dx =$

(A) $\frac{3}{5}$

(B) $\frac{2}{3}$

(C) 1

(D) $\frac{1}{3}$

(E) $\frac{8}{5}$

(4 pts) 5. The Simpson's rule approximation to $\int_0^{\pi} x \sin x dx$, with $n = 6$, is

$$\frac{\pi}{18}\left(0 + \frac{\pi}{3} + \frac{\sqrt{3}}{3}\pi + 2\pi + \frac{2\sqrt{3}}{3}\pi + \frac{5}{3}\pi + 0\right)$$

(A) True

(B) False

(4 pts) 6. The Midpoint rule approximation to $\int_0^2 \frac{x}{x+1}\, dx$, with $n = 4$, is $\frac{5}{6}$.

(A) True

(B) False

(10 pts) 7. Evaluate $\int_0^{\infty} x^2 e^{-x^3} dx$

(A) $\frac{1}{2}$

(B) 1

(C) the integral diverges

(D) $\frac{1}{3}$

(E) $\frac{1}{6}$

(10 pts) 8. The curve $y = \frac{x^2}{4} - \frac{\ln x}{2}$, $1 \le x \le 4$ is rotated about the x-axis. The area of the surface so generated is given by the integral:

(A) $2\pi \int_1^4 \left(\frac{x^2}{4} - \frac{\ln x}{2}\right) \sqrt{\frac{x^2}{4} - \frac{1}{4x^2}}\, dx$

(B) $2\pi \int_1^4 \left(\frac{x}{2} - \frac{1}{2x}\right) dx$

(C) $2\pi \int_1^4 \left(\frac{x^2}{4} - \frac{\ln x}{2}\right)\left(\frac{x}{2} - \frac{1}{2x}\right) dx$

(D) $2\pi \int_1^4 \left(\frac{x}{2} + \frac{1}{2x}\right) dx$

(E) $2\pi \int_1^4 \left(\frac{x^2}{4} - \frac{\ln x}{2}\right)\left(\frac{x}{2} + \frac{1}{2x}\right) dx$

(10 pts) 9. Find the x coordinate of the centroid of the region bounded by the curves, $y = e^x$, $y = 0$, $x = 0$, $x = 1$.

(A) $\bar{x} = e - 1$

(B) $\bar{x} = \frac{1}{e-1}$

(C) $\bar{x} = \frac{1}{e}$

(D) $\bar{x} = 1$

(E) $\bar{x} = \frac{e}{2}$

10. Evaluate the following limits. Provide justification for how you arrive at your answer.

(5 pts) (a) $\lim_{n\to\infty} \frac{\sqrt{3n^2+n}}{2n+1}$

(5 pts) (b) $\lim_{n\to\infty} \frac{e^n}{2^{n+1}}$

(6 pts) 11. Evaluate $\sum_{n=1}^{\infty} \frac{3^{n+1}}{5^n}$

(A) $\frac{3}{2}$

(B) $\frac{3}{5}$

(C) $\frac{9}{2}$

(D) $\frac{6}{5}$

(E) $\frac{5}{3}$

(4 pts) 12. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges

(A) True

(B) False