MATH 162 – SPRING 2004 – SECOND EXAM
MARCH 25, 2004

STUDENT NAME———————————————————————

STUDENT ID———————————————————————

RECITATION INSTRUCTOR———————————————————

RECITATION TIME———————————————————————

INSTRUCTIONS

1. Fill in your name, your student ID number, and your recitation instructor's name and recitation time above. Write your name, your student ID number and division and section number of your recitation section on your answer sheet, and fill in the corresponding circles.
2. Mark the letter of your response for each question on the mark-sense sheet.
3. There are 13 questions, each worth 8 points.
8. No books, notes or calculators may be used.

Useful formulas:

Moments and center of mass

$$M_x = \int_a^b \frac{1}{2}\left((f(x))^2 - (g(x))^2\right) dx, \quad M_y = \int_a^b x\,(f(x) - g(x))\,dx$$

$$\overline{x} = \frac{M_y}{M}, \quad \overline{y} = \frac{M_x}{M},$$

Arc length

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\ \ dx$$

Area of a surface of revolution

$$S = \int_a^b 2\pi f(x)\sqrt{1 + (f'(x))^2}\,dx$$

1) The mass of the region bounded by $f(x) = \frac{1}{2}\sqrt{4-2x^2}$, $g(x) = -\frac{1}{2}\sqrt{4-2x^2}$ and the y-axis is $\pi\sqrt{2}$. Its center of mass is

A) $(\frac{8}{3\pi\sqrt{2}}, 0)$

B) $(0, \frac{8}{3\pi\sqrt{2}})$

C) $(\frac{1}{2}, \frac{8}{3\pi\sqrt{2}})$

D) $(0, \frac{4}{3\pi\sqrt{2}})$

E) $(\frac{4}{3\pi\sqrt{2}}, 0)$

2) The improper integral

$$\int_0^1 \ln x \, dx =$$

A) $2\ln 2$

B) $-4\ln 2$

C) $2\ln 2$

D) -1

E) $-\frac{1}{9}$

3) The improper integral

$$\int_0^\infty x\, e^{-x^2}\ dx \quad \text{is equal to}$$

A) $\frac{1}{3}$

B) $\frac{1}{4}$

C) $\frac{1}{2}$

D) 1

E) 2

4) The length of the curve $y = \frac{x^3}{6} + \frac{1}{2x}$, $1 \leq x \leq 2$ is

A) $\frac{17}{12}$

B) $\frac{4}{3}$

C) $\frac{3}{2}$

D) 2

E) 1

5) The area of the surface obtained by rotating the curve

$$y = x^3, \quad 0 \leq x \leq 1$$

about the x-axis is

A) $\pi\sqrt{3}$

B) $\frac{2\pi}{3}$

C) $\frac{\pi}{27}(10\sqrt{10} - 1)$

D) $6\pi(3\sqrt{3} - 1)$

E) $\frac{\pi}{3}(10\sqrt{10} - 1)$

6) Find

$$\lim_{n\to\infty} \frac{\sqrt{2n^4 + n^2 + 1}}{2n^2 + 1}$$

A) $\frac{1}{2}$

B) 1

C) $\frac{\sqrt{2}}{2}$

D) 0

E) It does not exist.

7) The series

$$\sum_{n=1}^{\infty} \frac{\sqrt{2n^4+n^2+1}}{2n^2+1}$$

A) diverges

B) converges conditionally

C) converges by the ratio test

D) converges by the root test

E) converges by the integral test

8) Find

$$\lim_{n\to\infty} n \sin\left(\frac{1}{n}\right)$$

A) 0

B) 1

C) 2

D) ∞

E) it does not exist

9) The series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

A) converges by the ratio test

B) diverges by the ratio test

C) converges because $\lim_{n\to\infty} \sin(\frac{1}{n}) = 0$

D) converges by the limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

E) diverges by the limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

10)

$$\sum_{n=0}^{\infty} \frac{2^{n+1} - 3^n}{6^n} =$$

A) 1

B) 5

C) $\frac{7}{3}$

D) $\frac{1}{2}$

E) diverges

11) What is the smallest number of terms of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ that need to be added to compute its sum with error strictly less than 10^{-2}?

A) 3

B) 4

C) 5

D) 6

E) 7

12) Which of the following is a correct statement about the series

$$S_1 = \sum_{n=2}^{\infty} \frac{1}{n \ln n} \qquad \text{and} \qquad S_2 = \sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}?$$

A) S_1 and S_2 are divergent

B) S_1 converges but S_2 diverges

C) S_1 diverges but S_2 converges conditionally

D) S_1 converges and S_2 converges conditionally

E) S_1 and S_2 converge absolutely

13) Find the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{x^n}{n\ 3^n}.$$

A) $(-3, 3)$

B) $(-\frac{1}{3}, \frac{1}{3})$

C) $(-3, 3]$

D) $(-\frac{1}{3}, \frac{1}{3}]$

E) $[-3, 3)$