# MA162 — EXAM III — FALL 2016 — NOVEMBER 10, 2016
## TEST NUMBER 01

**INSTRUCTIONS:**

1. **Do not open the exam booklet until you are instructed to do so.**
2. **Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
3. **MARK YOUR TEST NUMBER ON YOUR SCANTRON**
4. Once you are allowed to open the exam, make sure you have a complete test. There are 7 different test pages (including this cover page).
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. **Circle your answers on this test booklet.**
6. Each problem is worth 8 points. Everyone gets 4 points. The maximum possible score is 100 points. No partial credit.
7. Do not leave the exam room during the first 20 minutes of the exam.
8. If you do not finish your exam in the first 50 minutes, you must wait until the end of the exam period to leave the room.
9. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

**DON'T BE A CHEATER:**

1. Do not give, seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only your instructor.
2. Do not look at the exam or scantron of another student.
3. Do not allow other students to look at your exam or your scantron.
4. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
5. Do not consult notes or books.
6. **Do not handle** phones or cameras, calculators or any electronic device until after you have finished your exam, handed it in to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

STUDENT ID NUMBER: ______________________________

SECTION NUMBER AND RECITATION INSTRUCTOR: ______________________________

**1.** Find all values of $p$ such that the series $\sum_{k=1}^{\infty} \sqrt{\frac{k^3+3k}{k^p+2}}$ converges.

A. $p > 1$

B. $p > 4$

C. $p > 5$

D. $p > 2$

E. $p > 7$

**2.** Which of the statements below is true for the series $\sum_{m=1}^{\infty} \frac{3^{-m}}{\sin(m)+2}$?

A. It diverges by comparison with $\sum_{m=1}^{\infty} \frac{1}{\sin(m)+2}$.

B. It diverges by comparison with $\sum_{m=1}^{\infty} 3^{-m}$.

C. It converges by comparison test $\sum_{m=1}^{\infty} \frac{1}{\sin(m)+2}$.

D. The comparison test is not applicable.

E. It converges by comparison with $\sum_{m=1}^{\infty} 3^{-m}$.

**3.** Which of the following series converge?

$$I.\ \sum_{k=1}^{\infty} \frac{1}{2k+1}; \quad II.\ \sum_{k=1}^{\infty} (-1)^{k-1} \frac{1}{\sqrt{k}+1}; \quad III.\ \sum_{k=1}^{\infty} \frac{1}{k^2+1}$$

A. All three

B. Only I and II

C. Only III

D. None

E. Only II and III

**4.** Consider $S = \sum_{m=1}^{\infty} (-1)^{m-1} \frac{1}{m^4}$ and its partial sum $S_n = \sum_{m=1}^{n} (-1)^{m-1} \frac{1}{m^4}$. According to the alternating series estimation theorem, what is the smallest $n$ such that $|S - S_n| \leq 16 \times 10^{-8}$?

A. $n = 101$

B. $n = 99$

C. $n = 51$

D. $n = 49$

E. $n = 499$

**5.** Which of the statements is true about the following series?

$$I.\ \sum_{n=1}^{\infty}(-1)^{n-1}\cos(n); \quad II.\ \sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{\ln(n+1)}; \quad III.\ \sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sqrt{n}+1}{n}.$$

A. All three are convergent

B. Only II is convergent

C. Only II and III are convergent

D. Only III is convergent

E. Only I and II are convergent

**6.** Which of the following statements is false?

A. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^p+n}$ is convergent for all $p>1$.

B. $\displaystyle\sum_{n=1}^{\infty} r^n$ diverges for all $r$ such that $|r|\geq 1$.

C. If $0\leq a_n\leq b_n$ and $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent, then $\displaystyle\sum_{n=1}^{\infty} b_n$ is also convergent.

D. If $0\leq a_n\leq b_n$ and $\displaystyle\sum_{n=1}^{\infty} a_n$ is divergent, then $\displaystyle\sum_{n=1}^{\infty} b_n$ is also divergent.

E. $\displaystyle\sum_{n=1}^{\infty}\frac{n^2}{r^n}$ converges for all $r$ such that $|r|>1$.

**7.** Which of the following series converge?

$$S_1 = \sum_{n=1}^{\infty} \frac{3^n}{4^n + 5^n}, \quad S_2 = \sum_{n=1}^{\infty} \frac{n^2}{n^3 + 5n}, \quad S_3 = \sum_{n=2}^{\infty} \frac{\ln n}{n},$$

A. $S_1$, $S_2$ and $S_3$

B. Only $S_1$ and $S_2$

C. Only $S_2$ and $S_3$

D. Only $S_1$ and $S_3$

E. Only $S_1$

**8.** Which of the following series converge?

$$S_1 = \sum_{n=1}^{\infty} \frac{n!}{10^n}, \quad S_2 = \sum_{n=1}^{\infty} \frac{10^n}{(n+1)^n}, \quad S_3 = \sum_{n=2}^{\infty} n^8\, 2^{-n},$$

A. $S_1$, $S_2$ and $S_3$

B. Only $S_1$ and $S_2$

C. Only $S_2$ and $S_3$

D. Only $S_1$ and $S_3$

E. Only $S_1$

**9.** Find the interval of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^n}{n\ 3^n}$.

A. $[-1, 4]$

B. $(-1, 4)$

C. $[-2, 4]$

D. $(-2, 4]$

E. $[-2, 4)$

**10.** Let $b_n > 0$ and $a_n$ be such that

$$a_{n+1} = (-1)^{n+1} \left(\frac{n+1}{2n+7}\right) a_n \text{ for } n \geq 1 \quad \text{and } \lim_{n\to\infty} \frac{|a_n|}{b_n} = 2.$$

We can say that the following are true:

A. $\displaystyle\sum_{n=1}^{\infty} a_n$ converges absolutely and $\displaystyle\sum_{n=1}^{\infty} b_n$ converges

B. $\displaystyle\sum_{n=1}^{\infty} a_n$ converges conditionally and $\displaystyle\sum_{n=1}^{\infty} b_n$ converges

C. $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges and $\displaystyle\sum_{n=1}^{\infty} b_n$ converges

D. $\displaystyle\sum_{n=1}^{\infty} a_n$ converges absolutely and $\displaystyle\sum_{n=1}^{\infty} b_n$ diverges

E. $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges and $\displaystyle\sum_{n=1}^{\infty} b_n$ diverges

**11.** The function $f(x) = \ln(2-x)$ is represented by the power series $\ln 2 - \sum_{n=1}^{\infty} a_n x^n$ where $a_n =$

A. $a_n = \dfrac{1}{2^n}$

B. $a_n = (-1)^n \dfrac{1}{n\ 2^n}$

C. $a_n = \dfrac{1}{n^2} 2^n$

D. $a_n = \dfrac{1}{n\ 2^n}$

E. $a_n = \dfrac{1}{n^2}$

**12.** Use that $\dfrac{2x}{(1-x^2)^2} = \dfrac{d}{dx}(\dfrac{1}{1-x^2})$ to conclude that $\dfrac{2x}{(1-x^2)^2} =$

A. $\sum_{n=1}^{\infty} 2n\ x^{2n-1}$

B. $\sum_{n=1}^{\infty} (-1)^n (2n+1)\ x^{2n+1}$

C. $\sum_{n=1}^{\infty} (2n-1)\ x^{2n+1}$

D. $\sum_{n=1}^{\infty} n\ x^{2n+1}$

E. $\sum_{n=1}^{\infty} \dfrac{x^{2n-1}}{n+1}$