

MA 16200
FINAL EXAM Form A (Test/Quiz Number 01)
May 8, 2015

NAME _____ YOUR TA'S NAME _____

STUDENT ID # _____ RECITATION TIME _____

1. You must use a #2 pencil on the mark-sense sheet (answer sheet).
2. If the cover of your question booklet is GREEN, write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below. If the cover is ORANGE, write 02 in the TEST/QUIZ NUMBER boxes and darken the spaces below.
3. On the mark-sense sheet, fill in your TA's name and the course number.
4. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.
5. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.
6. Sign the mark-sense sheet.
7. Fill in your name and your instructor's name on the question sheets above.
8. There are 25 questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1-25. Do all your work on the question sheets.
9. Turn in both the mark-sense sheets and the question sheets when you are finished.
10. If you finish the exam before 9:50, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 9:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.
11. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

1. Find a vector of length 6 that points in the opposite direction to $\langle 2, 1, -2 \rangle$

A. $\langle -2, -1, 2 \rangle$

B. $\langle 4, 2, -4 \rangle$

C. $\langle -6, -3, 6 \rangle$

D. $\langle -4, -2, 4 \rangle$

E. $\langle 6, 3, -6 \rangle$

2. Find the angle θ between $\vec{a} = \langle 4, 1, -1 \rangle$ and $\vec{b} = \langle -2, -2, -1 \rangle$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{3\pi}{4}$

D. $\frac{-\pi}{3}$

E. $\frac{\pi}{2}$

3. Find the area of the region enclosed by $y = x$ and $x = 4y - y^2$

- A. $\frac{7}{2}$
- B. $\frac{13}{6}$
- C. $\frac{11}{2}$
- D. $\frac{14}{3}$
- E. $\frac{9}{2}$

4. The base of a solid S is a triangular region with vertices at $(0, 0)$, $(1, 0)$, and $(1, 2)$. If the cross-sections perpendicular to the x -axis are squares, find the volume of the solid.

- A. $\frac{4}{3}$
- B. $\frac{2}{3}$
- C. 1
- D. $\frac{5}{3}$
- E. 2

5. A rectangular aquarium 2 feet long, 1 foot wide and 1 foot deep is filled with water. How much work is needed to pump the water out of the aquarium? Use the fact that water weighs 62.5 lb/ft^3

- A. 31.25 foot-pounds
- B. 62.5 foot pounds
- C. 93.75 foot pounds
- D. 125 foot pounds
- E. 156.25 foot pounds

6. Find the integral: $\int_1^e x \ln x \, dx$

- A. $\frac{1}{32}e^2 + \frac{1}{4}$
- B. $\frac{1}{16}e^2 + \frac{1}{4}$
- C. $\frac{1}{8}e^2 + \frac{1}{4}$
- D. $\frac{1}{4}e^2 + \frac{1}{4}$
- E. $\frac{1}{2}e^2 + \frac{1}{4}$

7. Find the integral: $\int_0^{\pi/4} (\cos^2 x - \sin^2 x) dx$

A. $-\frac{\sqrt{2}}{2}$

B. $-\frac{1}{2}$

C. 0

D. $\frac{1}{2}$

E. $\frac{\sqrt{2}}{2}$

8. What does the integral $\int \frac{x^3}{\sqrt{1+x^2}} dx$ become after a trigonometric substitution?

A. $\int \frac{\sin^3 \theta}{\cos^2 \theta} d\theta$

B. $\int \frac{\sin^2 \theta}{\cos^3 \theta} d\theta$

C. $\int \frac{\sin^2 \theta}{\cos^4 \theta} d\theta$

D. $\int \frac{\sin^3 \theta}{\cos^3 \theta} d\theta$

E. $\int \frac{\sin^3 \theta}{\cos^4 \theta} d\theta$

9. What does the integral $\int x^2\sqrt{x^2-1} dx$ become after a trigonometric substitution?

(a) $\int \frac{\sin^3 \theta}{\cos^3 \theta} d\theta$

(b) $\int \frac{\sin^2 \theta}{\cos^4 \theta} d\theta$

(c) $\int \frac{\sin^2 \theta}{\cos^5 \theta} d\theta$

(d) $\int \frac{\sin^3 \theta}{\cos^4 \theta} d\theta$

(e) $\int \frac{\sin^3 \theta}{\cos^5 \theta} d\theta$

10. Compute $\int \frac{1}{x^2+x} dx =$

A. $\frac{1}{2} \tan^{-1} \left(x + \frac{1}{2} \right) + C$

B. $\ln |x^2 + x| + C$

C. $\ln \left| \frac{x+1}{x} \right| + C$

D. $\ln \left| \frac{x}{x+1} \right| + C$

E. $\frac{1}{2} \tan^{-1} (x) + C$

11. From the Table of Integrals in the book we know

$$\int \frac{u \, du}{\sqrt{1+u}} = \frac{2}{3}(u-2)\sqrt{1+u} + C$$

So what is $\int_0^{3/2} \frac{x \, dx}{\sqrt{1+2x}}$ equal to?

- A. $1/2$
- B. $3/2$
- C. $2/3$
- D. $2/\sqrt{3}$
- E. $3(1 + \sqrt{3})$

12. Find all values of p for which the integral $\int_0^\infty e^{(p-1)x} \, dx$ converges.

- A. $p < 1$
- B. $p < 0$
- C. $p > 1$
- D. $p > 0$
- E. The integral diverges for all p

13. The derivative of a function g is $g'(x) = \sqrt{\sec^4 x - 1}$. What is the length of the curve $y = g(x)$ on the interval $0 \leq x \leq \frac{\pi}{4}$?

- A. 1
- B. 2
- C. 4
- D. $\sqrt{3}$
- E. $\sqrt{2}$

14. The region bounded by the curves $y = x^2$, $y = 0$, and $x = 2$ has area $\frac{8}{3}$. If its centroid is at (\bar{x}, \bar{y}) , then $\bar{y} =$

- A. $3/2$
- B. $6/5$
- C. $3/5$
- D. $3/4$
- E. 2

15. Which of the following sequences converge?

I. $a_n = \frac{\sqrt[3]{n^4 + 2}}{2n + 1}$

II. $a_n = \frac{(-1)^n n^2}{2 + 3n^2}$

III. $a_n = \frac{(\ln n)^2}{n}$

A. None of them

B. Only III

C. II and III

D. I and II

E. I and III

16. Compute

$$\sum_{n=1}^{\infty} \frac{2^{n-1} + (-2)^n}{3^n}$$

A. $\frac{3}{5}$

B. $\frac{-3}{5}$

C. $\frac{-1}{3}$

D. $\frac{1}{3}$

E. $\frac{1}{5}$

17. Which of these series converge?

I. $\sum_{n=1}^{\infty} \frac{\sqrt{n^3 + 1}}{\sqrt[3]{n^9 + 7}}$

II. $\sum_{n=1}^{\infty} \frac{\sqrt{n^3 + 1}}{n^{5/2}}$

III. $\sum_{n=1}^{\infty} \frac{100^n}{n!}$

- A. I only
- B. II only
- C. III only
- D. I and II only
- E. I and III only

18. Find the interval of convergence of

$$\sum_{n=1}^{\infty} \frac{(-3)^n}{n\sqrt{n}} x^n$$

- A. $\left[0, \frac{1}{3}\right]$
- B. $\left(-\frac{1}{3}, \frac{1}{3}\right)$
- C. $\left[-\frac{1}{3}, \frac{1}{3}\right)$
- D. $\left(-\frac{1}{3}, \frac{1}{3}\right]$
- E. $\left[-\frac{1}{3}, \frac{1}{3}\right]$

19. The first 3 terms of the Maclaurin series of $f(x) = \frac{x}{1+x^2}$ are

- A. $x + x^3 + x^5$
- B. $x - x^3 + x^5$
- C. $1 - x^2 + x^4$
- D. $1 + x^2 + x^4$
- E. $-x - x^3 - x^5$

20. Knowing that the Maclaurin series of $\ln(1+x)$ is given by

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n},$$

find the smallest number of terms of the series that one needs to add to compute $\ln(1.1)$ with an error less than or equal to 10^{-8} .

- A. 2
- B. 3
- C. 5
- D. 9
- E. 7

21. If the Maclaurin series of a function $f(x)$ is

$$\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{3n(n+6)},$$

then $f^{(6)}(0)$ is equal to

- A. $5/3$
- B. $-15/6$
- C. $10/3$
- D. $9/7$
- E. $8/5$

22. The binomial series is

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!} x^2 + \frac{k(k-1)(k-2)}{3!} x^3 + \dots$$

What is the Maclaurin series of $f(x) = \frac{1}{(1+x)^3}$?

- A. $\sum_{n=0}^{\infty} \frac{(-1)^n (n+2)(n+1)}{2} x^n$
- B. $\sum_{n=0}^{\infty} \frac{(-1)^n (n+1)n}{2} x^n$
- C. $\sum_{n=0}^{\infty} \frac{(-1)^n (n+2)(n+1)}{2} x^{n+1}$
- D. $\sum_{n=0}^{\infty} \frac{(-1)^n (n+1)n}{2} x^{n+1}$
- E. $\sum_{n=1}^{\infty} \frac{(-1)^n (n+2)(n+1)}{2} x^{n-1}$

23. Find the foci of the ellipse $11(x - 1)^2 + 7(y - 3)^2 = 77$.

- A. $(1, 1), (1, 5)$
- B. $(0, -2), (0, 2)$
- C. $(1, -5), (1, -1)$
- D. $(2, 0), (-2, 0)$
- E. $(-1, -2), (-1, 2)$

24. The curve whose equation in polar coordinates is $r = 4 \sec \theta$ is

- A. a circle
- B. a line
- C. a parabola
- D. a cardioid
- E. an ellipse

25. Which of the following is an argument for the polar form of $2\sqrt{3} - 2i$?

A. $11\pi/6$

B. $5\pi/3$

C. $7\pi/6$

D. $2\pi/3$

E. $4\pi/3$