MA 16500
EXAM 2 INSTRUCTIONS
VERSION 01
October 14, 2019

Your name _____ Your TA's name _____

Student ID # _____ Section # and recitation time _____

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name (NOT the lecturer's name)** and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit **SECTION NUMBER**.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the space provided for each of the questions 1–12. While **mark all your work on the scantron sheet**, you should **show your work on the exam booklet**. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.

8. There are 12 questions, each worth 8 points. The maximum possible score is

$8 \times 12 + 4$ (for taking the exam) $= 100$ points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

Questions

1. Find the following values:

(a) The derivative of $f(x) = \dfrac{e^x}{\sqrt{x^2+1}}$ at $x = 1$.

(b) The derivative of $g(x) = \ln(\sin(x^2))$ at $x = \sqrt{\dfrac{\pi}{2}}$.

A. (a) $f'(1) = \dfrac{e}{2\sqrt{2}}$ (b) $g'\left(\sqrt{\dfrac{\pi}{2}}\right) = 0$

B. (a) $f'(1) = \dfrac{e}{\sqrt{2}}$ (b) $g'\left(\sqrt{\dfrac{\pi}{2}}\right) = 0$

C. (a) $f'(1) = \dfrac{e}{2\sqrt{2}}$ (b) $g'\left(\sqrt{\dfrac{\pi}{2}}\right) = 2\sqrt{\dfrac{\pi}{2}}$

D. (a) $f'(1) = \dfrac{e}{\sqrt{2}}$ (b) $g'\left(\sqrt{\dfrac{\pi}{2}}\right)$ DNE

E. (a) $f'(1) = \sqrt{2}e$ (b) $g'\left(\sqrt{\dfrac{\pi}{2}}\right)$ DNE

2. Find the following values:

(a) The derivative of $f(x) = \sin^{-1}\left(\frac{1}{x}\right)$ at $x = 2$.

(b) The derivative of $g(x) = \tan^{-1}(\sqrt{x})$ at $x = 3$.

A. (a) $f'(2) = -\frac{1}{2\sqrt{3}}$ (b) $g'(3) = \frac{1}{4\sqrt{3}}$

B. (a) $f'(2) = \frac{1}{2\sqrt{3}}$ (b) $g'(3) = \frac{1}{2\sqrt{3}}$

C. (a) $f'(2) = -\frac{1}{2\sqrt{3}}$ (b) $g'(3) = \frac{1}{8\sqrt{3}}$

D. (a) $f'(2) = \frac{1}{2\sqrt{3}}$ (b) $g'(3) = \frac{1}{\sqrt{3}}$

E. (a) $f'(2) = 2\sqrt{3}$ (b) $g'(3) = \frac{1}{2}$

3. A person walks along a straight path, and a sensor is placed at the point P which is 2 meters away from the closest point Q on the path.

Let x be the distance between the person and the point Q, and let θ be the angle formed by the line PQ and the line connecting the sensor and the person. (See the picture below.)

Compute $\dfrac{d\theta}{dx}$.

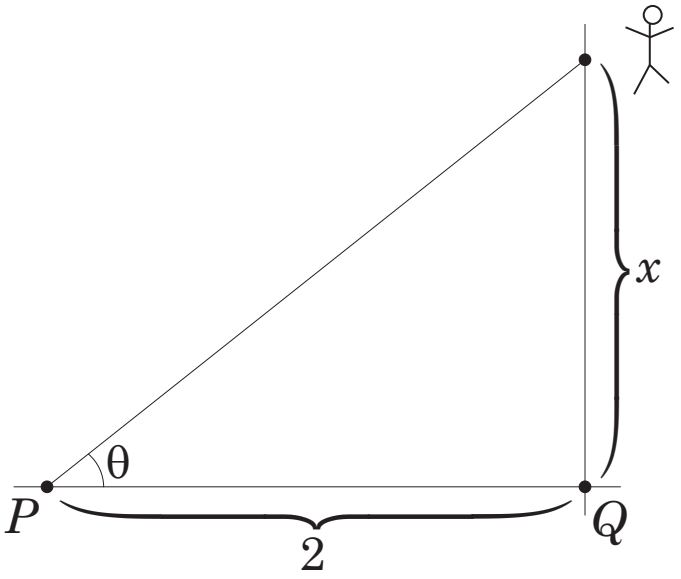
A. $\dfrac{d\theta}{dx} = \dfrac{1}{1+x^2}$

B. $\dfrac{d\theta}{dx} = \dfrac{4}{4+x^2}$

C. $\dfrac{d\theta}{dx} = \dfrac{2}{4+x^2}$

D. $\dfrac{d\theta}{dx} = \dfrac{4}{\sqrt{4-x^2}}$

E. $\dfrac{d\theta}{dx} = \dfrac{2}{\sqrt{4-x^2}}$



4. The slope of the tangent line to the graph of the curve defined by

$$\ln(x^2 - 3y) = x - y - 1$$

at the point $(2, 1)$ is:

A. 0

B. 1/2

C. 3/4

D. 1

E. 3/2

5. Find all such values of $b > 0$ that the graph of $y = b^x$ and the graph of $y = b^{-x}$ intersect perpendicularly.

A. $b = 2$ and $b = 1/2$

B. $b = e$ and $b = 1/e$

C. $b = e$ ONLY

D. $B = 1/e$ ONLY

E. There is no such value b.

6. Find the derivative of

$$f(x) = 64\sqrt{1+\sqrt{1+\sqrt{x}}}$$

at $x = 64$.

A. 1/2

B. 1/3

C. 1/4

D. 1/6

E. 1/8

7. Compute the following limit:

$$\lim_{h \to 0} \frac{(1+2h)^{5+3h} - 1}{h}.$$

A. 6

B. $e^{5/2}$

C. $e^{2/3}$

D. 10

E. 1

8. Consider the function

$$y = f(x) = x^3 + x^2 + 4x + 1.$$

It is one-to-one, and hence has its inverse function f^{-1}.

Observe that the point $(1, 7)$ is on the graph of $y = f(x)$.

Find the equation of the tangent line to the graph of $y = f^{-1}(x)$ at the point $(7, 1)$.

A. $y - 1 = \dfrac{1}{3x^2 + 2x + 4}(x - 7)$

B. $y = \dfrac{1}{9}x + \dfrac{2}{9}$

C. $y = -\dfrac{1}{9}x + \dfrac{16}{9}$

D. $y = 9x - 62$

E. We cannot determine the equation, because we do not have the explicit formula for $f^{-1}(x)$.

9. The length of a rectangle is increasing at a rate of 8 cm/sec and its width is increasing at a rate of 3 cm/sec.

When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing ?

A. 100 cm^2/sec

B. 190 cm^2/sec

C. 140 cm^2/sec

D. 24 cm^2/sec

E. 48 cm^2/sec

10. A conical tank is 20 ft deep and the radius at the top is 15 ft. If the water is leaking from the tank at the rate of 72 ft^3/min, how fast is the level of the water dropping when the water is 12 ft deep ?

HINT: The volume V of a circular cone of height h and base radius r is given by $V = \frac{1}{3}\pi r^2 h$.

A. $\frac{2}{3\pi}$ ft/min

B. $\frac{24}{25\pi}$ ft/min

C. $\frac{8}{3\pi}$ ft/min

D. $\frac{8}{27\pi}$ ft/min

E. $\frac{8}{9\pi}$ ft/min

11. A ladder 5 ft long is leaning against a wall with its upper end moving down along the wall at a rate of 1/2 ft/sec.

How fast is the lower end of the ladder moving away from the wall when its distance to the wall is 3 ft ?

A. 1/3 ft/sec

B. 2/3 ft/sec

C. 1 ft/sec

D. 3/2 ft/sec

E. 4/3 ft/sec

12. Once Kate's kite reaches a height of 50 ft (above her hand), it rises no higher and drifts due east at the speed of 5 ft/sec.

What is the rate of change of angle of elevation at the moment when she has released 100 ft of string ?

A. $-\frac{1}{150}$ rad/sec

B. $-\frac{1}{40}$ rad/sec

C. $-\frac{1}{25}$ rad/sec

D. $-\frac{1}{5}$ rad/sec

E. $-\frac{\pi}{5}$ rad/sec

Note: The minus sign in front of the numbers above indicates that the angle of elevation is decreasing at the time.