NAME
STUDENT ID
RECITATION INSTRUCTOR
RECITATION TIME

DIRECTIONS
1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

(8) 1. Find the absolute maximum and absolute minimum values of the function
     \[ f(x) = \frac{x^2}{x^4+1} \]
     on the interval \([1, 2]\).

     \[
     \text{abs. max. } f( ) = \\
     \text{abs. min. } f( ) =
     \]

(4) 2. Explain why the function \( f(x) = x^{2/3} \) does not satisfy the hypotheses of the Mean Value Theorem on the interval \([-2, 3]\).

(8) 3. Use calculus to find a positive number such that the sum of the number and its reciprocal is as small as possible.
(20) 4. Find each of the following as a real number, +∞, −∞ or write DNE (does not exist).

(a) \[ \lim_{x \to 0} \frac{e^x + e^{-x} - 2}{x^2} \]

(b) \[ \lim_{x \to 0} \frac{\sin^{-1} x}{x} \]

(c) \[ \lim_{x \to \infty} (x - \sqrt{x^2 - 1}) \]

(d) \[ \lim_{x \to 0^+} (1 + x)^{1/x} \]

(14) 5. The numbers 3 and −1 are critical numbers of the function
\[ f(x) = 2x^5 - 5x^4 - 10x^3. \]
Showing all necessary work, decide whether \( f \) has a local maximum or a local minimum.

(a) at 3 using the first derivative test,

loc.

(b) at −1 using the second derivative test.

loc.
6. Let \( f(x) = \frac{1}{1 + e^{-x}} \). Give all the requested information and sketch the graph of the function on the axes below. Give both coordinates of the intercepts, local extrema and points of inflection, and give an equation for each asymptote. Write NONE where appropriate.

![Graph of function](image)

<table>
<thead>
<tr>
<th>Domain</th>
<th>Intercepts</th>
<th>Symmetry</th>
<th>Horizontal Asymptotes</th>
<th>Vertical Asymptotes</th>
<th>Intervals of Increase</th>
<th>Intervals of Decrease</th>
<th>Local Maxima</th>
<th>Local Minima</th>
<th>Intervals of Concave Down</th>
<th>Intervals of Concave Up</th>
<th>Points of Inflection</th>
</tr>
</thead>
</table>
7. A conical drinking cup is made from a circular piece of paper of radius $R$ by cutting out a sector and joining the edges CA and CB. Find the maximum capacity of such a cup.

8. Find the most general antiderivative of $f(x) = 5e^x - \frac{1}{1 + x^2}$.

9. Find $f(x)$ if $f'(x) = 2\sin x + \sec^2 x$ and $f(0) = 3$.

10. If $\int_{2}^{8} f(x)dx = 1.7$ and $\int_{5}^{8} f(x)dx = 2.5$, find $\int_{2}^{5} f(x)dx$. 