MA 16500
EXAM 3 INSTRUCTIONS
VERSION 01
November 8, 2016

Your name ____________________ Your TA's name ____________________

Student ID # ________________ Section # and recitation time ____________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your question booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit SECTION NUMBER.

6. Sign the scantron sheet. All the answers should be marked on the scantron sheet.

7. Blacken your choice of the correct answer in the spaces provided for each of the questions 1–12. Do all your work on the question sheets. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

8. There are 12 questions, each worth 8 points. The maximum possible score is

$8 \times 12 + 4$ (for taking the exam) $= 100$ points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheets and the exam booklets.

11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheets and exam booklets.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

Questions

1. Consider the function $f(x) = \dfrac{x}{\sqrt{x-4}}$ on the closed interval $[5, 12]$.

Find

(a) the absolute maximum, and

(b) the absolute minimum.

A. (a) $3\sqrt{2}$ (b) 4

B. (a) 5 (b) 4

C. (a) 5 (b) $3\sqrt{2}$

D. (a) 5 (b) DNE

E. (a) $3\sqrt{2}$ (b) 5

2. Consider the function $f(x) = (x+2)^6(x-1)^7$ on the interval $(-\infty, \infty)$.

Choose the correct statement from below.

A. The function f takes its local maximum at $x = -2$ and its local minimum at $x = -\dfrac{8}{13}$.

B. The function f takes its local maximum at $x = -2$ and its local minima at $x = -\dfrac{8}{13}$ and $x = 1$.

C. The function f takes its local minimum at $x = -2$ and its local maximum at $x = -\dfrac{8}{13}$.

D. The function f takes its local maximum at $x = -2$ and its local minimum at $x = 1$.

E. The function f takes its local minimum at $x = -\frac{8}{13}$, but there is no local maximum.

3. Compute the following limits

$$\text{(a)} \lim_{x\to\infty} \frac{x^2 - \ln\left(\frac{2}{x}\right)}{3x^2+2x}.$$

$$\text{(b)} \lim_{x\to\infty} x \cdot \left(e^{\frac{1}{x}} - 1\right).$$

A. (a) $-\frac{2}{3}$ (b) 1

B. (a) $-\frac{2}{3}$ (b) 0

C. (a) $\frac{1}{3}$ (b) 1

D. (a) $\frac{1}{3}$ (b) 0

E. (a) $\frac{1}{3}$ (b) ∞

4. Consider the function $f(x) = x^{\frac{2}{3}}$ on the interval $[-1, 1]$.

Choose the correct statement from below.

A. Since $f(1) = f(-1) = 1$, by Rolle's Theorem, there exists $c \in (-1, 1)$ such that $f'(c) = 0$.

B. Even though $f(1) = f(-1) = 1$, there does not exist any value $c \in (-1, 1)$ such that $f'(c) = 0$. This gives a counter-example to the Rolle's Theorem in case when the function involves a fractional power.

C. Even though $f(1) = f(-1) = 1$, there does not exist any value $c \in (-1, 1)$ such that $f'(c) = 0$. This is because f is not continuous at $x = 0$ and it does not contradict the Rolle's Theorem.

D. Even though $f(1) = f(-1) = 1$, there does not exist any value $c \in (-1, 1)$ such that $f'(c) = 0$. This is because f is not differentiable at $x = 0$ and it does not contradict the Rolle's Theorem.

E. Even though $f(1) = f(-1) = 1$, there does not exist any value $c \in (-1, 1)$ such that $f'(c) = 0$. We observe $f'(0) = +\infty$. In order for the Rolle's Theorem to hold, the derivatives at the open interval $(-1, 1)$ have to be all finite numbers, and this is why the conclusion of the Rolle's Theiorem fails here.

5. Compute the following limit

$$\lim_{x\to\infty}\left(\frac{x-1}{x+1}\right)^{5x}.$$

A. e^{10}

B. e^{-10}

C. e^{5}

D. e^{-2}

E. 1

6. Choose the picture from below that best describes the graph of the function

$$f(x) = \frac{x}{x^2 - 9}.$$

A.

B.

C.

D.

E.

7. A corollary to the Mean Value Theorem says: if a continuous function f on $[a, b]$ is such that $f'(x) = 0$ for all x over (a, b), then f is constant on $[a, b]$.

We would like to use this corollary to evaluate $\tan^{-1}(2) + \cot^{-1}(2)$.

Choose the correct statement from below:

A. The function $f(x) = \tan^{-1}(x) + \cot^{-1}(x)$ is such that $f'(x) = 0$ on $(1, 2)$, and hence it is constant on $[1, 2]$.
Therefore,

$$\tan^{-1}(2) + \cot^{-1}(2) = f(2) = f(1) = \tan^{-1}(1) + \cot^{-1}(1) = \frac{\pi}{2}.$$

B. The function $f(x) = \tan^{-1}(x) + \cot^{-1}(x)$ is such that $f'(x) = 0$ on $(1, 2)$, and hence it is constant on $[1, 2]$.
Therefore,

$$\tan^{-1}(2) + \cot^{-1}(2) = f(2) = f(1) = \tan^{-1}(1) + \cot^{-1}(1) = \frac{\pi}{4}.$$

C. The function $f(x) = \tan^{-1}(x) + \cot^{-1}(x)$ does not have the property that $f'(x) = 0$ on $(1, 2)$, and hence we cannot apply the corollary to compute the value of $\tan^{-1}(2) + \cot^{-1}(2)$.

D. The value of $\tan^{-1}(2)$ is not any of the usual nice angles $\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}$, or $\frac{\pi}{6}$.
The value of $\cot^{-1}(2)$ is not, either.
Therefore, its sum is also none of the usual nice angles $\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}$.

E. The tangent function $\tan(x)$ and the cotangent function $\cot(x)$ are reciprocal to each other, and hence so are their inverse functions. So we are thinking about the value of the form $a + \frac{1}{a}$, which varies as the value a varies, and not a constant. Therefore, the above corollary does not apply to our situation.

8. Find the interval where the curve

$$y = \frac{2x^2}{x^2 - 1}$$

is concave up.

A. $(-1, 1)$

B. $(-\infty, -1) \cup (1, \infty)$

C. $(-\infty, -1)$

D. $(1, \infty)$

E. $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

9. Find the equation of the slant asymptote to the graph of the function

$$f(x) = \frac{6x^3 + 3x^2}{3x^2 + 1}.$$

A. $y = x$

B. $y = 2x$

C. $y = 6x + 3$

D. $y = x + 2$

E. $y = 2x + 1$

10. What two positive real numbers whose product is 50 have the smallest sum ?

A. $10\sqrt{2}$ & $\frac{5}{2}\sqrt{2}$

B. 10 & 5

C. $5\sqrt{2}$ & $5\sqrt{2}$

D. 25 & 2

E. 50 & 1

11. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths 3 cm and 8 cm if two sides of the rectangle lie along the legs.

A. 4 cm^2

B. 5 cm^2

C. 6 cm^2

D. 7 cm^2

E. 8 cm^2

12. Find the equation of the line through the point $(2, 5)$ that cuts off the least area from the first quadrant.

A. $y = -x + 7$

B. $y = -2x + 9$

C. $y = -\frac{5}{2} + 10$

D. $y = -\frac{3}{2} + 8$

E. $y = -\frac{1}{2} + 6$