MA 16500
EXAM 3 INSTRUCTIONS
VERSION 01
November 13, 2018

Your name ____________________ Your TA's name ____________________

Student ID # ________________ Section # and recitation time __________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name (NOT the lecturer's name)** and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit **SECTION NUMBER**.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the space provided for each of the questions 1–12. While mark all your work on the scantron sheet, you should show your work on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.

8. There are 12 questions, each worth 8 points. The maximum possible score is

$8 \times 12 + 4$ (for taking the exam) $= 100$ points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ____

STUDENT SIGNATURE: ____

Questions

1. Consider the function $f(x) = x - \sqrt{x}$ on the interval $[0, 4]$.

Let M be the absolute maximum value of $f(x)$ on $[0, 4]$ and let m be the absolute minimum value of $f(x)$ on $[0, 4]$.

Compute $M + m$.

A. $\frac{5}{4}$

B. $\frac{7}{4}$

C. $\frac{5}{2} - \frac{1}{\sqrt{2}}$

D. 2

E. $\frac{5}{2}$

2. What is the number of local minima and local maxima combined together of the function

$$f(x) = x^4(x-2)^3$$

on the interval $[-1, 3]$?

A. 0

B. 1

C. 2

D. 3

E. 4

WARNING: We follow the convention of the textbook: We exclude the end points of the interval from the consideration of local minimum and local maximum.

3. How many inflection points does the graph of the function

$$y = f(x) = x^5 - 5x^4 + 25x$$

have ?

A. 0

B. 1

C. 2

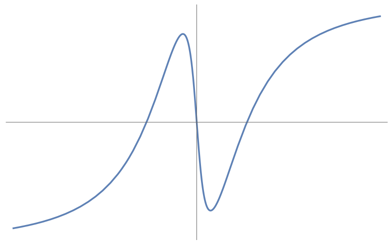
D. 3

E. 4

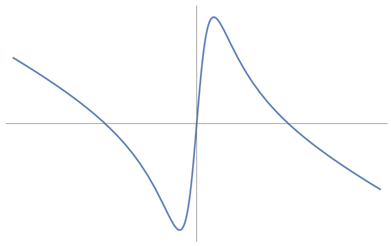
4. Which of the following best describes the graph of

$$y = f(x) = \frac{x}{x^2 + 1}?$$

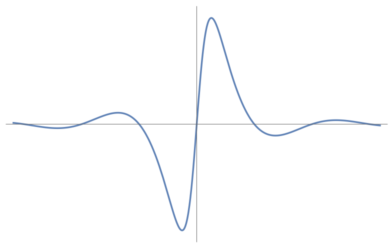
A.



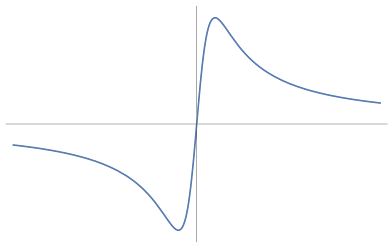
B.



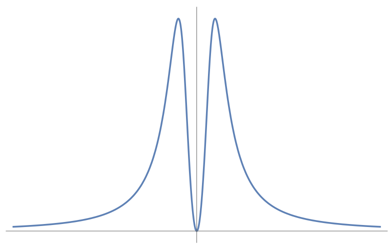
C.



D.



E.



5. Compute the following limit

$$\lim_{x \to (\pi/2)^-} \left(x - \frac{\pi}{2}\right) \cdot \tan x.$$

A. $\dfrac{\pi}{2}$

B. ∞

C. 0

D. 1

E. -1

6. Compute the following limit

$$\lim_{x\to\infty}\left(\frac{x+1}{x-1}\right)^x.$$

A. 1

B. e^{-2}

C. e^2

D. 2

E. ∞

7. Consider the function

$$f(x) = 2 - \sqrt{|x|}$$

over the interval $[-4, 1]$.

Choose the correct statement from below.

A. Since f is continuous on $[-4, 1]$, differentiable over $(-4, 1)$, and since $\dfrac{f(1) - f(-4)}{1 - (-4)} = 1/5$, the Mean Value Theorem says there exists $c \in (-4, 1)$ such that $f'(c) = 1/5$. In fact, we can take $c = \dfrac{25}{4}$.

B. Since f is continuous on $[-4, 1]$, differentiable over $(-4, 1)$, and since $\dfrac{f(1) - f(-4)}{1 - (-4)} = 1/5$, the Mean Value Theorem says there exists $c \in (-4, 1)$ such that $f'(c) = 1/5$. In fact, we can take $c = -\dfrac{25}{4}$.

C. Even though f is continuous on $[-4, 1]$, f is not differentiable over $(-4, 1)$. We compute $\dfrac{f(1) - f(-4)}{1 - (-4)} = 1/5$. Therefore, as the negation of the Mean Value Theorem we conclude that there does not exist any $c \in (-4, 1)$ such that $f'(c) = 1/5$.

D. Even though f is continuous on $[-4, 1]$, f is not differentiable over $(-4, 1)$. So we cannot apply the Mean Value Theorem to this situation.

E. Even though f is continuous on $[-4, 1]$, f is not differentiable over $(-4, 1)$. We compute $\dfrac{f(1) - f(-4)}{1 - (-4)} = 1/5$. But we still observe that $f'(c) = 1/5$ for $c = -\dfrac{25}{4} \in (-4, 1)$. So we conclude that the Mean Value Theorem is false in this case.

8. Compute the following limits:

$$(a) \quad \lim_{x\to 0} \frac{x - \sin x}{x - \tan x} \qquad\qquad (b) \quad \lim_{x\to 0} \frac{\cos x}{x+1}$$

A. (a) $-\frac{1}{2}$ (b) 1

B. (a) $-\frac{1}{2}$ (b) 0

C. (a) DNE (b) 1

D. (a) DNE (b) 0

E. (a) -1 (b) 0

9. Consider the following function

$$y = f(x) = \ln\left(\frac{x^2 + 2x + 1}{x^2 + 2x - 15}\right).$$

Choose the correct description of the horizontal and vertical asymptotes of the graph of the function from below.

A. The graph has 1 horizontal, and 2 vertical asymptotes.

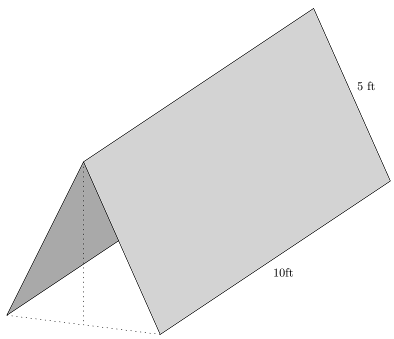
B. The graph has 1 horizontal, and 3 vertical asymptotes.

C. The graph has 2 horizontal, and 3 vertical asymptotes.

D. The graph has 2 horizontal, and 2 vertical asymptotes.

E. The graph has 0 horizontal, and 2 vertical asymptotes.

10. A 10ft by 10ft tarp is lifted in the center to form a triangular shelter. What is the maximum volume that can be covered under the shelter in this way?



A. $62.5\ \text{ft}^3$

B. $120\ \text{ft}^3$

C. $125\ \text{ft}^3$

D. $240\ \text{ft}^3$

E. $250\ \text{ft}^3$

11. Two wooden bars of equal length AO = BO = 1 ft are connected by a hinge at point O so that one can rotate the bar BO around as shown in the picture below.

Find the rotation angle θ $(0 < \theta < \pi)$ which maximizes the area of the triangle $\triangle$ABC.

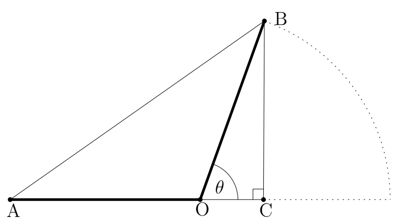
A. $\dfrac{\pi}{6}$

B. $\dfrac{\pi}{4}$

C. $\dfrac{\pi}{3}$

D. $\dfrac{\pi}{2}$

E. $\dfrac{2\pi}{3}$



12. A cardboard box with NO TOP has such a shape that the bottom is a rectangle whose length is twice as long as its width.

We want the box to hold the volume of $\dfrac{32}{3}$ m^3.

What should the height of this rectangular box be, if we want to minimize the amount of cardboard to make the box ?

A. $\dfrac{2}{3}$ m

B. $\dfrac{4}{3}$ m

C. $\sqrt{2}$ m

D. $\dfrac{4\sqrt{2}}{3}$ m

E. $\dfrac{2}{3^{1/3}}$ m