MA 16500
FINAL EXAM INSTRUCTIONS
VERSION 01
DECEMBER 16, 2016

Your name ______________ Your TA's name ______________

Student ID # ______________ Section # and recitation time ______________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your question booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit SECTION NUMBER.

6. Sign the scantron sheet. All the answers should be marked on the scantron sheet.

7. Blacken your choice of the correct answer in the spaces provided for each of the questions 1–25. Do all your work on the question sheets. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

8. There are 25 questions, each worth 8 points. The maximum possible score is
$8 \times 25 = 200$ points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheets and the question booklets.

11. If you finish the exam before 9:55, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 9:55, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ____

STUDENT SIGNATURE: ____

Questions

1. Find the domain of definition for the function $f(x) = \sqrt[4]{\dfrac{x}{x+1} \cdot (x-1)^3}$.

A. $(-1, 0] \cup [1, \infty)$
B. $(-1, 0) \cup (1, \infty)$
C. $(-\infty, -1)$
D. $(-\infty, -1) \cup [0, 1]$
E. $(-\infty, \infty)$

2. How many solutions are there for the equation

$$2\cos(x) = \sin(x) + 1$$

over the interval $[0, 2\pi]$?

A. 0
B. 1
C. 2
D. 3
E. 4

3. Evaluate

$$\lim_{x \to 0} \frac{x - \sin(x)}{x - \tan(x)}.$$

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. 0

D. ∞

E. The limit does not exist.

4. Consider the function $f(x) = \tan(x)^{\sin(x)}$.

Compute $f'\left(\frac{\pi}{4}\right)$.

A. 2

B. 1

C. $\sqrt{2}$

D. 0

E. $\ln 2$

5. Let $f(x)$ be the function defined by

$$f(x) = \begin{cases} \dfrac{x^2 + a}{x + 1} & \text{if} \quad x < -1 \\ x^3 + b & \text{if} \quad x \geq -1. \end{cases}$$

Determine the values of a and b so that f is continuous over $(-\infty, \infty)$.

A. $a = -1, b = -1$

B. $a = -1, b = 1$

C. $a = -1, b = 2$

D. $a = -1, b = 3$

E. There are no such values for a and b.

6. The function f satisfies the condition

$$f(2) = 5 \text{ and } f'(2) = -3.$$

If g is the inverse function of f, then find $g(5)$ and $g'(5)$.

A. $g(5) = 2$ and $g'(5) = 3$

B. $g(5) = 2$ and $g'(5) = -3$

C. $g(5) = 2$ and $g'(5) = \frac{1}{3}$

D. $g(5) = 2$ and $g'(5) = -\frac{1}{3}$

E. $g(5) = 2$, but we cannot determine $g'(5)$ from the given information.

7. Find the slope of the tangent line to the curve defined by the equation

$$x^3y - 3x + y^2 = -1$$

at the point $(x, y) = (1, 1)$.

A. 2

B. 1

C. 0

D. -1

E. -2

8. Find the algebraic expression for $\sec(\tan^{-1}(x))$.

A. $\dfrac{1}{1+x^2}$

B. $-\dfrac{1}{1+x^2}$

C. $\sqrt{1+x^2}$

D. $\dfrac{1}{\sqrt{1+x^2}}$

E. $\dfrac{1}{\sqrt{1-x^2}}$

9. Compute

$$\lim_{x\to\infty}\left(\sqrt{x^2+4x+1}-x\right)$$

A. -2

B. 0

C. 2

D. ∞

E. The limit does not exist.

10. Use the linear approximation of the function $f(x)=\sqrt{x^2+x}$ at $a=1$ to estimate the value $f(1.1)$.

A. $\sqrt{2}+\dfrac{3}{2\sqrt{2}}$

B. $\dfrac{0.3}{2\sqrt{2}}$

C. $\dfrac{5.3}{2\sqrt{2}}$

D. $\dfrac{4.1}{2\sqrt{2}}$

E. $\dfrac{4.3}{2\sqrt{2}}$

11. Find the absolute maximum a and absolute minimum b of the function

$$f(x) = 2\cos(x) + \sin(2x)$$

on the closed interval $[0, \pi/2]$

A. $a = 2,\ b = \frac{3\sqrt{3}}{2}$

B. $a = 2,\ b = 0$

C. $a = 2,\ b = -1$

D. $a = \frac{3\sqrt{3}}{2},\ b = 0$

E. $a = \frac{3\sqrt{3}}{2},\ b = 2$

12. Find the interval where the function $f(x) = 4x^3 - 6x^2 + 3x$ takes the value 2.

A. $(-1, 0)$

B. $(0, 1)$

C. $(1, 2)$

D. $(2, 3)$

E. $(3, 4)$

13. Evaluate

$$\int_{-2}^{0} x|x+1|dx$$

A. 2

B. 1

C. 0

D. -1

E. -2

14. Let $y = f(x)$ be a function defined over $(-\infty, \infty)$, whose derivative is given by the formula

$$f'(x) = (x-1)^2(x+2)(x-5).$$

Choose the right statement from below.

A. $f(1)$ and $f(5)$ are local maxima, $f(-2)$ is a local minimum of the function $y = f(x)$.

B. $f(1)$ and $f(5)$ are local minima, $f(-2)$ is a local maximum of the function $y = f(x)$.

C. $f(-2)$ is a local maximum, $f(5)$ is a local minimum, and $f(1)$ is neither a local maximum nor a local minimum of the function $y = f(x)$.

D. $f(-2)$ is a local minimum, $f(5)$ is a local maximum, and $f(1)$ is neither a local minimum nor a local maximum of the function $y = f(x)$.

E. $f(-2)$ is a local maximum, $f(5)$ is a local minimum of the function $y = f(x)$. But we cannot decide whether $f(1)$ is a local maximum or a local minimum from the given information.

15. Compute the limit

$$\lim_{x\to\infty} x^{\ln(2)/(\ln(x)+1)}.$$

A. 0

B. 2

C. ∞

D. $\ln(2)$

E. $\ln(2)+1$

16. Compute the limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{3n} \cdot \sin\left(i \cdot \frac{\pi}{3n}\right).$$

A. ∞

B. 0

C. $\dfrac{1}{2}$

D. 1

E. π

17. In a right triangle, leg x is increasing at a rate of 2 m/s, while leg y is decreasing at a rate that keeps the area of the triangle constant at 6 m^2.

What is the rate of the change of the hypoteneuse when $x = 3$ m ?

A. $-\dfrac{7}{15}$

B. $\dfrac{7}{15}$

C. $-\dfrac{14}{15}$

D. $\dfrac{14}{15}$

E. $\dfrac{28}{15}$

18. Find the x-coordinate of the point on the graph of the function $y = \sqrt{x}$ which is closest to the given fixed point $\left(\dfrac{3}{2}, 0\right)$.

A. 3

B. 2

C. 1

D. $\sqrt{2}$

E. $\sqrt{\dfrac{1}{2}}$

19. Compute $\int_0^2 \frac{x^3}{\sqrt{x^2+1}}dx$.

A. 0

B. $\frac{4}{3}(\sqrt{5}+1)$

C. $\frac{2}{3}(\sqrt{5}+1)$

D. $\frac{2}{3}(\sqrt{5}-1)$

E. $\frac{2}{3}\sqrt{5}$

20. The half-life of Radium 226 is 1590 years.

What is the value of the relative growth rate k, i.e., the constant k such that the formula for the mass as a function of time (in years) is given by $m(t) = m(0)e^{kt}$?

A. $\frac{1590}{\sqrt{2}}$

B. $\frac{1590}{\ln 2}$

C. $\frac{\ln 2}{1590}$

D. $-\frac{\ln 2}{1590}$

E. $\frac{1590}{\ln\left(\frac{1}{2}\right)}$

21. Choose the one which describes best the graph of the function

$$f(x) = \frac{x}{x^2+9} \text{ over the interval } (-\infty, \infty).$$

A.

B.

C.

D.

E.

22. Set $f(x) = \displaystyle\int_{\tan(x)}^{4} \cos^2(x)dx.$

Find the formula for $f'(x)$.

A. $\cos^2(\tan(x))$

B. $-\cos^2(\tan(x))$

C. $\cos^2(\tan(x))\sec^2(x)$

D. $-\cos^2(\tan(x))\sec^2(x)$

E. $-2\cos(x)\sin(x)\sec^2(x)$

23. A particle moves along a straight line, satisfying the conditions

$$\begin{cases} a(t) &= 2t-4, \\ v(0) &= 3. \end{cases}$$

What is the total distance traveled from $t=0$ to $t=2$?

A. 3

B. 2

C. $\dfrac{2}{3}$

D. $\dfrac{1}{3}$

E. Since the initial condition $s(0)$ for the position function s is not given, we cannot determine s. Accordingly, we cannot compute the total distance traveled from the given information.

24. An ellipse has foci $(\pm 4-1, 7) = (3, 7)$ & $(-5, 7)$, and it passes through the points $(-1, 10)$ and $(-1, 4)$. Which of the following is the equation of the ellipse?

A. $\dfrac{(x+1)^2}{9} + \dfrac{(y-7)^2}{25} = 1$

B. $\dfrac{(x-1)^2}{9} + \dfrac{(y+7)^2}{25} = 1$

C. $\dfrac{(x+1)^2}{25} + \dfrac{(y-7)^2}{9} = 1$

D. $\dfrac{(x-1)^2}{25} + \dfrac{(y+7)^2}{9} = 1$

E. $\dfrac{(x+1)^2}{16} + \dfrac{(y-7)^2}{9} = 1$

25. Which of the following is an asymptote of the hyperbola

$$9x^2 - 4y^2 - 36x - 8y - 4 = 0 \ ?$$

A. $3x - 2y - 8 = 0$

B. $2x - 3y - 8 = 0$

C. $3x - 2y = 0$

D. $2x - 3y - 4 = 0$

E. $3x + 2y - 8 = 0$