MA 16500
FINAL EXAM VERSION 01
December 15, 2022

INSTRUCTIONS

Your name ____________________ Your TA's name ____________________

Student ID # _______________ Section # and recitation time __________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name, i.e., the name of your recitation instructor (NOT the lecturer's name)** and the course number.

4. Fill in your **NAME** and **PURDUE ID NUMBER**, and blacken in the appropriate spaces. Put 00 at the front of PUID to make it a 10 digit number, and then fill it in.

5. Fill in the four-digit **SECTION NUMBER**. Your section number is a 3 digit number. Put 0 at the front to make it a 4 digit number, and then fill it in.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the space provided for each of the questions 1–25. While mark all your answers on the scantron sheet, you should **show your work** on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.

8. There are 25 questions, each of which is worth 8 points. The maximum possible score is

25 questions $\times$ 8 points $=$ 200 points.

9. **NO calculators, electronic device, books, or papers are allowed.** Use the back of the test pages for scrap paper.

10. After you finish the exam, **turn in BOTH the scantron sheet and the exam booklet**.

11. If you finish the exam before 12:25 PM, you may leave the room after turning in the scantron sheet and the exam booklet. **If you don't finish before 12:25 PM, you should REMAIN SEATED** until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. There is no individual seating. Just follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs/proctors will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor/proctor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

Questions

1. Evaluate the following definite integral

$$\int_0^{10} |x-1|\, dx.$$

A. -40

B. 25

C. 40

D. 41

E. 45

2. (8 points) Compute

$$\int_0^{\pi/2} \frac{\cos(t)}{1+\sin^2(t)}\,dt.$$

A. 1

B. $\dfrac{1}{\sqrt{2}}$

C. $\ln(2)$

D. $\dfrac{\pi}{4}$

E. 0

3. (8 points) Compute the limit

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{1}{1+\frac{2k}{n}} \right) \frac{1}{n}.$$

HINT: Identify the sum as an approximation for a definite integral (i.e., a Riemann Sum), and then compute the limit as the definite integral.

A. $\ln(3)$

B. $\frac{1}{2}\ln(2)$

C. $\ln(3) - \ln(2)$

D. $\frac{1}{2}\ln(3)$

E. $2\ln(3)$

4. (8 points) Compute

$$\frac{d}{dx}\left(\int_{e^{-x}}^{e^{x}} \ln(t)\,dt\right).$$

A. $2x$

B. $e^x - e^{-x}$

C. $x(e^x - e^{-x})$

D. $x(e^x + e^{-x})$

E. $2xe^x$

5. The half-life of cesium-137 is 30 years. Suppose we have a 60-mg sample at the beginning.

How long will it take until the remain of the sample becomes 1-mg?

A. 60 years

B. 90 years

C. 180 years

D. $30\left(2+\dfrac{\ln 15}{\ln 2}\right)$ years

E. $30(2+\ln 15)$ years

6. A snowball melts so that its surface decreases at the rate of 2 cm^2/min. How fast is the volume decreasing when the radius is 8 cm ?

HINT: The surface area S and the volume V of a sphere of radius r are given as follows:

$$\begin{cases} S = 4\pi r^2, \\ V = \dfrac{4}{3}\pi r^3. \end{cases}$$

A. 4 cm^3/min

B. $\dfrac{16}{3}$ cm^3/min

C. $\dfrac{8}{3}$ cm^3/min

D. 16 cm^3/min

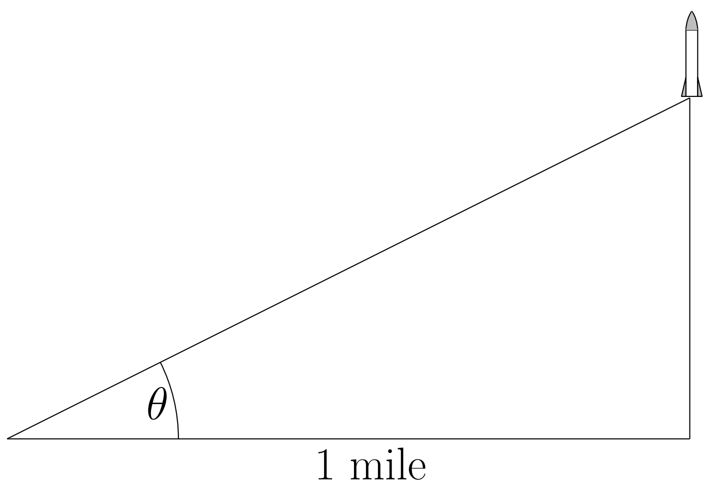
E. 8 cm^3/min

7. (8 points) Suppose you are recording a rocket launch from a location 1 mile away from the launch. When the rocket is $\frac{1}{2}$ mile high, you must rotate your camera angle at a rate of $\frac{1}{9}$ radians per second to keep the rocket in frame.

How fast is the rocket moving (in miles per hour) at that moment ?

HINT: The unit conversion reads as follows:

$$\frac{1}{9}\text{ rad/sec} = 400\text{ rad/hour}$$



A. $50\sqrt{2}$ miles/hour

B. $200\sqrt{5}$ miles/hour

C. 320 miles/hour

D. 500 miles/hour

E. 1000 miles/hour

8. (8 points) Find the real numbers a and b so that the following function becomes continuous over $(-\infty, \infty)$:

$$f(x) = \begin{cases} \dfrac{x^2 + ax - 3}{x - 3} & \text{if} \quad x < 3 \\ x^2 + b & \text{if} \quad x \geq 3. \end{cases}$$

A. $a = -2$, $b = -5$

B. $a = -2$, $b = -7$

C. $a = 0$, $b = -9$

D. $a = 2$, $b = 4$

E. There are no real numbers a and b that make the function continuous over $(-\infty, \infty)$.

9. Use the linear approximation for $f(x) = \sqrt[3]{x}$ at $a = 8$ to estimate $\sqrt[3]{7.76}$.

A. 1.94

B. 1.96

C. 1.98

D. 2.00

E. 2.02

10. (8 points) Find the equation of the tangent line to the curve

$$\sin(xy) = \ln(x)$$

at the point $(1, \pi)$.

A. $y = -\dfrac{x}{\pi} + \pi + \dfrac{1}{\pi}$

B. $y = -\dfrac{\pi}{2}x + \dfrac{3\pi}{2}$

C. $y = -(\pi + 1)x + 2\pi + 1$

D. $y = -(\pi + 2)x + 2\pi + 2$

E. $y = -\pi x + 2\pi$

11. Find a formula for the inverse of the function

$$f(x) = \frac{1 - \ln x}{1 + \ln x} \quad \text{for} \quad x > 1.$$

A. $f^{-1}(x) = \dfrac{1 - x}{1 + x}$

B. $f^{-1}(x) = \dfrac{1 + \ln x}{1 - \ln x}$

C. $f^{-1}(x) = e^{\frac{1+x}{1-x}}$

D. $f^{-1}(x) = e^{\frac{x-1}{x+1}}$

E. $f^{-1}(x) = e^{\frac{1-x}{1+x}}$

12. (8 points) The function is given by the formula

$$f(x) = 2^{\sqrt{x^2+\ln(x)}}.$$

Compute $f'(1)$.

A. $3\ln(2)$

B. $\ln(2)$

C. 3

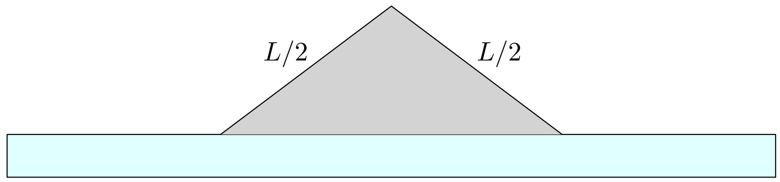
D. $\frac{3}{2}$

E. $-2\ln(2)$

13. (8 points) A farmer wants to fence in a region along a river.

The fenced in area will be in the shape of an isoceles triangle as shown in the diagram below.

(No fencing will be needed along the river, which is the blue part of the picture. The two sides of the triangle with fencing have equal length.)



If the farmer has L feet of fencing, what is the maximum area that he can fence in like this ?

A. $\dfrac{L^2}{8}$ square feet

B. $\dfrac{L^2}{2}$ square feet

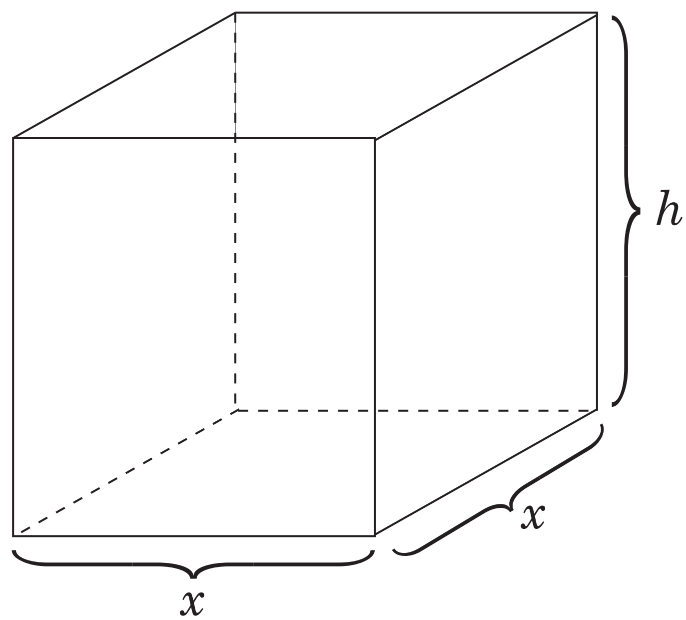
C. $\dfrac{L^2\sqrt{3}}{16}$ square feet

D. $\dfrac{L^2}{16}$ square feet

E. $\dfrac{L^2\sqrt{3}}{8}$ square feet

14. (8 points) A container is to be made in the shape of a box with an open top and a square base. The material for the square base costs \$3/ft^2 and the material for the sides costs \$2/ft^2.

If the container is to have a volume of 6 ft^3, what should be the length of each side of the base in order to minimize the cost ?



A. 3 ft

B. $3\sqrt[3]{2}$ ft

C. $\sqrt[3]{6}$ ft

D. $\sqrt{2}$ ft

E. 2 ft

15. The derivative of a function f is given by

$$f'(x) = (x-1)x(x+1)^2(x+2)$$

The function f has a local minimum only at:

A. $x = -2$ and $x = 1$

B. $x = -2$

C. $x = -2$ and $x = 0$

D. $x = -1$ and $x = 0$

E. $x = -1$ and $x = 1$

16. (8 points) Find the absolute minimum and absolute maximum of the function

$$f(x) = \frac{\sin(x)}{2 - \cos(x)} \quad \text{on the interval } [0, \pi].$$

A. absolute minimum $= 0$, absolute maximum $= 1/2$.

B. absolute minimum $= -1/2$, absolute maximum $= 1/\sqrt{3}$.

C. absolute minimum $= 0$, absolute maximum $= 1/\sqrt{3}$.

D. absolute minimum $= -1/2$, absolute maximum $= 1/2$.

E. absolute minimum $= 0$, absolute maximum $= \sqrt{3}/2$.

17. (8 points) Compute the following limit

$$\lim_{x\to\infty}\left[x\cdot\ln\left(\frac{2\tan^{-1}(x)}{\pi}\right)\right].$$

A. $-\pi$

B. $-\dfrac{2}{\pi}$

C. $\dfrac{\pi}{2}$

D. $-\dfrac{1}{\pi}$

E. ∞

18. Find all solutions to

$$3\cos^2(x) - \sin^2(x) = 0 \text{ with } 0 \leq x \leq 2\pi.$$

A. $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

B. $x = \frac{5\pi}{6}, \frac{7\pi}{6}$

C. $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

D. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

E. There are no solutions with $0 \leq x \leq 2\pi$

19. (8 points) Consider the function $f(x) = x^{\ln x}$.

Evaluate $f'(e)$.

A. $1/e$

B. 1

C. e

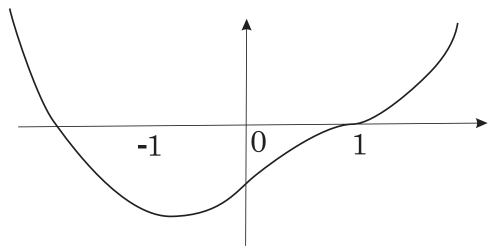
D. e^e

E. 2

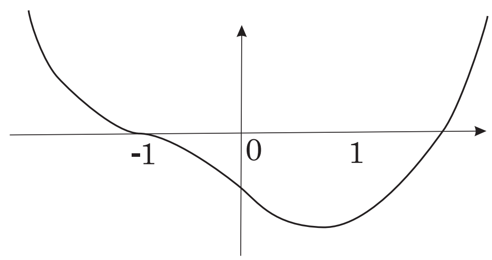
20. (8 points) Which of the following graphs can be the graph of a function $y = f(x)$ whose 2nd derivative is given by the formula

$$f''(x) = -x(x+1)^2(x-1)\ ?$$

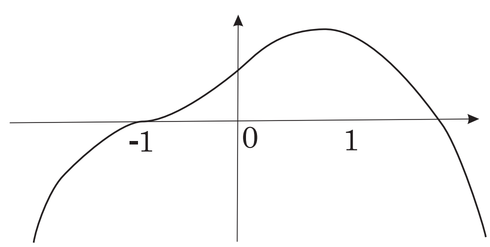
A.



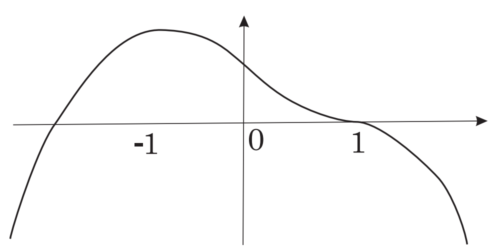
B.



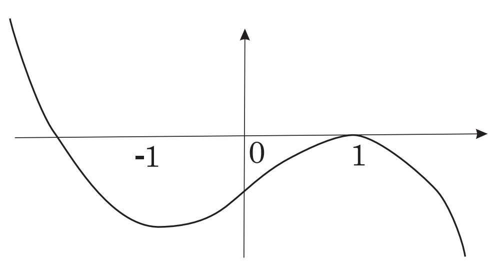
C.



D.



E.



21. (8 points) The position of a particle is given as a function of time t in seconds denoted by $f(t)$.

Its 1st derivative, called the velocity v, is given by the formula

$$v = f'(t) = 6(t-1)(t-3).$$

Find the total distance traveled during the first 4 seconds.

A. 8

B. 16

C. 24

D. 32

E. We need the initial position $f(0)$, which is missing, to determine $f(t)$. Since we cannot determine the position function $f(t)$, we cannot compute the total distance traveled.

22. (8 points) Compute the following limit.

$$\lim_{x\to\infty}\left(\frac{5x+1}{5x-1}\right)^{2x+1}.$$

A. $e^{4/5}$

B. e

C. $e^{5/4}$

D. e^2

E. e^5

23. (8 points) Compute the following limits.

(i) $\lim_{x\to 1} \dfrac{x^3 - 1}{\sqrt{x} - 1}$

(ii) $\lim_{x\to 0} \left[x \cdot \sin\left(\dfrac{1}{x}\right)\right]$

A. (i) 3/2 (ii) 1

B. (i) DNE (ii) 0

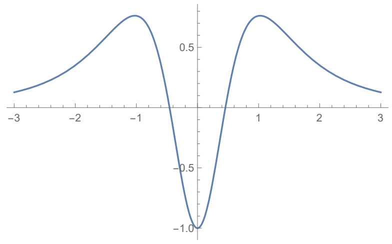
C. (i) DNE (ii) 1

D. (i) 6 (ii) DNE

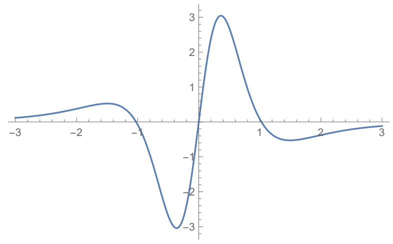
E. (i) 6 (ii) 0

24. The graphs of $y = f'(x)$ and $y = f''(x)$ are given below:

$y = f'(x)$

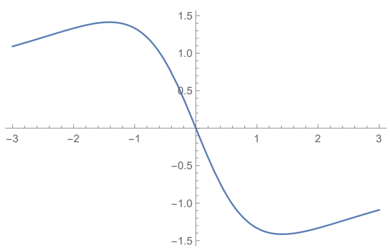


$y = f''(x)$

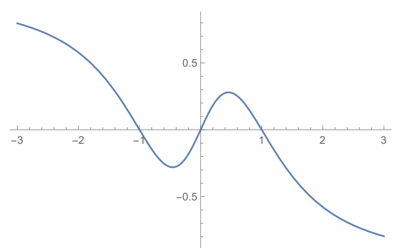


Which of the following then can be the graph of $y = f(x)$?

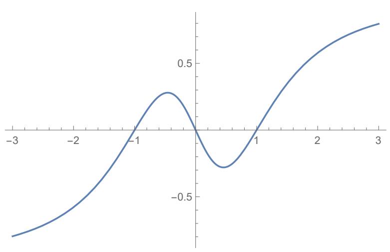
A.



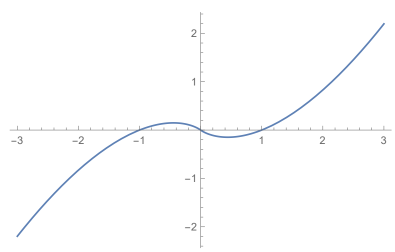
B.



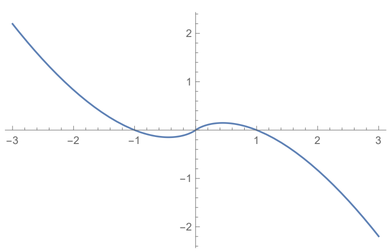
C.



D.



E.



25. Consider the function $f(x) = x(x^4 - 1)$ over the interval $[0, 2]$.

How many values of $c \in (0, 2)$ satisfy the statement of the Mean Value Theorem

$$f'(c) = \frac{f(2) - f(0)}{2 - 0} \ ?$$

A. 4

B. 3

C. 2

D. 1

E. 0