DIRECTIONS

1. Write your name, 10-digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3, and 4.

2. The test has four (4) pages, including this one.

3. Write your answers in the boxes provided.

4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.

5. Credit for each problem is given in parentheses in the left hand margin.

6. No books, notes or calculators may be used on this test.

(10) 1. Let \( \vec{a}, \vec{b}, \vec{c} \) be three-dimensional vectors. For each statement below, circle T if the statement is always true, or F if it is not always true.

   (i) \( \vec{a} \cdot \vec{a} = |\vec{a}|^2 \) \hspace{1cm} T \hspace{0.5cm} F

   (ii) \( \vec{a} \cdot (\vec{b} \cdot \vec{c}) = (\vec{a} \cdot \vec{b}) \cdot \vec{c} \) \hspace{1cm} T \hspace{0.5cm} F

   (iii) \( \vec{a} \cdot (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) \cdot \vec{c} \) \hspace{1cm} T \hspace{0.5cm} F

   (iv) \( \vec{a} \times \vec{b} = -\vec{b} \times \vec{a} \) \hspace{1cm} T \hspace{0.5cm} F

   (v) If \( \vec{a} \times \vec{c} = \vec{b} \times \vec{c} \), then \( \vec{a} = \vec{b} \) \hspace{1cm} T \hspace{0.5cm} F

(6) 2. For what values of \( b \) are the vectors \( <2, -1, b> \) and \( <b^2, 3, b> \) orthogonal?

   \[ b = \]

(4) 3. Find \( \vec{a} \cdot \vec{b} \) if \( |\vec{a}| = 3 \), \( |\vec{b}| = 6 \) and the angle between \( \vec{a} \) and \( \vec{b} \) is \( \frac{\pi}{3} \) radians.

   \[ \vec{a} \cdot \vec{b} = \]
4. Find a vector that has the same direction as $<-2, 4, 2>$ but has length 6.

5. Are the vectors $<1, -2, 3>$ and $<3, -6, 9>$ orthogonal, parallel or neither?

6. Consider the three points $A(1, 1, 1), B(2, 0, 2), C(1, 1, 2)$.
   (a) Find $\vec{AB} \times \vec{AC}$

   $$\vec{AB} \times \vec{AC} =$$

   (b) Find the area of the triangle with vertices $A, B, C$.

   (c) Find a unit vector orthogonal to the plane that passes through the points $A, B, C$. 

7. Find the area of the region bounded by the curves

\[ y = \sqrt{x + 2}, \quad y = \frac{1}{x + 1}, \quad x = 0, \quad x = 2. \]

8. Let \( R \) be the region bounded by \( y = x, \ y = 2 - x, \) and \( y = 0. \)
   
   (a) Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating \( R \) about the line \( x = 5, \) using the method of disks/washers.

(b) Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating \( R \) about the line \( y = 2, \) using the method of cylindrical shells.
9. The base of a solid is a triangular region with vertices \( A(0,0), \ B(1,0), \) and \( C(0,1) \). Cross-sections perpendicular to the \( y \)-axis are semicircles. Find the volume of the solid.

\[
V =
\]

10. Find the average value of \( f(x) = \sqrt{x} \) on the interval \([0, 4]\).

11. Evaluate the integrals

(a) \( \int x^3 \ln x \, dx \)

(b) \( \int_0^\pi t \sin 3t \, dt \)