NAME _____

STUDENT ID _____

RECITATION INSTRUCTOR _____

RECITATION TIME _____

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Page 4	/36
TOTAL	/100

DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3, and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this test.

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(14)   1. Circle the letter of the correct response. (You need not show work for this problem).

(a) Which of the following statements are true for any series $\sum\limits_{n=1}^{\infty} a_n$?

(I) If $\sum\limits_{n=1}^{\infty} a_n$ is absolutely convergent, then $\sum\limits_{n=1}^{\infty} a_n$ is convergent.

(II) If $\lim\limits_{n\to\infty} |a_n| = 0$, then $\sum\limits_{n=1}^{\infty} a_n$ is convergent.

(III) If $\lim\limits_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = 1$, then $\sum\limits_{n=1}^{\infty} a_n$ is divergent.

A.   (I) only   B.   (I) and (II) only   C.   (II) only   D.   (III) only   E.   none

(b) Which of the following series converge?

(I) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n-1}}{n^p}$, with any $p > 0$.

(II) $\sum\limits_{n=1}^{\infty} \dfrac{n^2 + 1}{3n^3 - 2n}$

(III) $\sum\limits_{n=1}^{\infty} \dfrac{\sin(2n)}{n^3}$

A.   (I) only   B.   (II) only   C.   (III) only   D.   (I) and (III) only   E.   all

2. Determine whether each series is convergent or divergent. You must verify that the conditions of the test are satisfied and write your conclusion in the small box.

(8) (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)}$

Show all necessary work here:

By the _____ test, the series is

(8) (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt{n}}{n+4}$

Show all necessary work here:

By the _____ test, the series is
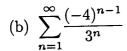
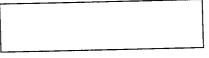(8) (c) $\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$

Justify your answer and show all necessary work here:

The series is

(8) 3. Find the sum of the series if it is convergent or write "divergent" in the box. No partial credit.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4^n}$

(b) $\sum_{n=1}^{\infty} \frac{(-4)^{n-1}}{3^n}$

(8) 4. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!}$.

(a) Write out the first five terms of the series.

(b) Find the smallest number of terms that we need to add in order to estimate the sum of the series with error $< \frac{1}{20}$.

terms

(10) 5. Determine whether the series $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$ is absolutely convergent, conditionally convergent, or divergent. You must justify your answer.

The series is

(16) 6. For the power series $\sum_{n=1}^{\infty}(-1)^n n 4^n x^n$, find the following, showing all work.

(a) The radius of convergence R.

$$\boxed{R = }$$

(b) The interval of convergence. (Don't forget to check the end points).

$$\boxed{\text{Intervals of convergence}}$$

(10) 7. (a) Find the power series representation of $-\dfrac{1}{5-x}$ (about $a = 0$).

$$\boxed{-\dfrac{1}{5-x} = \sum}$$

(b) Use integration of power series and the fact that $\ln(5-x) = -\int \dfrac{1}{5-x}\, dx$ for a certain value of the constant of integration C, to find the power series for $\ln(5-x)$.

$$\boxed{\ln(5-x) = \sum}$$

(10) 8. Find the Taylor series for $f(x) = \dfrac{1}{x}$ centered at $a = 2$.

$$\boxed{\dfrac{1}{x} = \sum}$$