MA 16600
EXAM 3 INSTRUCTIONS
VERSION 01
April 11, 2023

Your name _________________________ Your TA's name _________________________

Student ID # ___________________ Section # and recitation time ____________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name, i.e., the name of your recitation instructor (NOT the lecturer's name)** and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit **SECTION NUMBER**. Your section number is a 3 digit number. Put 0 at the front to make it a 4 digit number, and then fill it in.

6. **Sign the scantron sheet.**

7. Blacken your choice of the correct answer in the space provided for each of the questions 1–12. While mark all your answers on the scantron sheet, you should show your work on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.

8. There are 12 questions, 10 of which are worth 8 points and 2 of which are worth 10 points. The maximum possible score is

10 questions $\times$ 8 points $+$ 2 questions $\times$ 10 points $=$ 100 points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. There is no individual seating. Just follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs/proctors will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor/proctor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ________________________________

STUDENT SIGNATURE: ________________________________

Questions

1. (8 points) Compute the following series

$$\sum_{k=0}^{\infty} \frac{5^{k+1} + (-3)^k}{6^k}.$$

A. $30 + \dfrac{2}{3}$

B. $25 - \dfrac{1}{3}$

C. $6 + \dfrac{2}{3}$

D. 6

E. The series is divergent as a whole, since, while the first part is convergent, the second part oscillates and hence is divergent.

2. (8 points) Compute the following series

$$\sum_{k=1}^{\infty} \frac{3}{2k(k+2)}.$$

A. 9

B. 9/2

C. 2

D. 1

E. ∞

3. (8 points) Consider the series

$$\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} \left(\frac{1}{k^2 + 2k + 3} \right) \sin(k).$$

We conclude

A. the series is convergent since $\lim_{k\to\infty} a_k = 0$.

B. the series is divergent by Ratio Test since $\lim_{k\to\infty} \left| \frac{a_{k+1}}{a_k} \right| = 1$.

C. the series is absolutely convergent by Comparison Test with $\sum_{k=1}^{\infty} \frac{1}{k^2}$.

D. the series is convergent by Limit Comparison Test with $\sum_{k=1}^{\infty} \frac{1}{k^2}$.

E. the series is divergent by Test for Divergence since $\lim_{k\to\infty} a_k$ does not exist.

4. (8 points) Which of the following series converge ?

I. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k\{\ln(k)\}^2}$

II. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k\ln(k)}$

III. $\displaystyle\sum_{k=1}^{\infty}(-1)^k \ln\left(\frac{1}{k}\right)$

IV. $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{k\ln(k)}$

A. III and IV only.

B. II and IV only.

C. I, II and IV only.

D. I and IV only.

E. IV only.

5. (8 points) Which of the following statements are always true ?

I. $\lim\limits_{k\to\infty}(k\cdot a_k)=2$, then $\displaystyle\sum_{k=1}^{\infty} a_k$ diverges.

II. If $\displaystyle\sum_{k=1}^{\infty} |a_k|$ converges, then $\displaystyle\sum_{k=1}^{\infty} \frac{a_k}{k}$ converges.

III. If the series $\displaystyle\sum_{k=1}^{\infty} a_k$ converges with $a_k > 0$ and $\lim_{k\to\infty} \dfrac{b_k}{a_k} = 0$, then the series $\displaystyle\sum_{k=1}^{\infty} b_k$ converges.

IV. If $0 < b_k < c_k$ for all k, and $\sum_{k=1}^{\infty} c_k$ converges, then $\displaystyle\sum_{k=1}^{\infty}(-1)^k b_k$ converges conditionally.

A. I and II only.

B. II and IV only.

C. II and III only.

D. I and IV only.

E. I, II and III only.

6. Which of the following series converge ?

I. $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}k^{\frac{1}{k}}$

II. $\displaystyle\sum_{k=1}^{\infty}\frac{\cos(\pi k)}{5k+3}$

III. $\displaystyle\sum_{k=1}^{\infty}\left(\frac{1}{3^k}-\frac{1}{3^{k+1}}\right)$

IV. $\displaystyle\sum_{k=1}^{\infty}\frac{k!}{k^k}$

A. II and IV only.

B. I and IV only.

C. I, II and III only.

D. II and III only.

E. II, III and IV only.

7. (8 points) The series

$$\sum_{k=2}^{\infty} \frac{k^2}{(1+k^5)^p}$$

is convergent if and only if:

A. $p < 3/5$.

B. $p \geq 3/5$.

C. $p > 3/5$.

D. $p < 1$.

E. $p > 1$.

8. (8 points) Find the interval of convergence I for the power series

$$\sum_{k=2}^{\infty} \frac{(4x-3)^k}{2^{k+1}\ln(k)}.$$

A. $I = \left[\frac{1}{4}, \frac{5}{4}\right)$

B. $I = \left(\frac{1}{4}, \frac{5}{4}\right]$

C. $I = \left(\frac{1}{4}, \frac{5}{4}\right)$

D. $I = [1, 5)$

E. $I = (1, 5]$

9. (10 points) Given that

$$\arctan(x^3) = \int_0^x \frac{3t^2}{1+t^6}dt,$$

find the Maclaurin series of the function $\arctan(x^3)$ for $|x| < 1$.

HINT: $\dfrac{1}{1-z} = \displaystyle\sum_{k=0}^{\infty} z^k$. What would you like to plug in for z ?

A. $\arctan(x^3) = \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} x^{6k+3}$

B. $\arctan(x^3) = \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{6k+3} x^{6k+3}$

C. $\arctan(x^3) = \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{6k+3} x^{6k+2}$

D. $\arctan(x^3) = \displaystyle\sum_{k=0}^{\infty} \frac{3(-1)^k}{6k+1} x^{6k+1}$

E. $\arctan(x^3) = \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{k+1} x^{6k}$

10. (8 points) We compute the Taylor series for $f(x) = \dfrac{1}{x^4}$ centered at $a = 10$.

What is the coefficient of $(x-10)^2$ in the Taylor series ?

A. $\dfrac{2!}{10^5}$

B. $-\dfrac{10^5}{2!}$

C. $-\dfrac{1}{2!10^5}$

D. $\dfrac{1}{10^5}$

E. $\dfrac{1}{2!10^6}$

11. (10 points) Compute value of the series

$$\sum_{k=0}^{\infty} \frac{k+1}{2^k}.$$

HINT: We know

$$f(x) = \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + \cdots .$$

Compute the power series for $f'(x)$.

A. 2

B. 4

C. $\ln 2$

D. $2 \ln 2$

E. The series diverges.

12. (8 points) The Alternating Series Test shows that

$$\sum_{k=1}^{\infty}(-1)^k \frac{3}{7k}$$

converges to a value S.

Set

$$S_N = \sum_{k=1}^{N}(-1)^k \frac{3}{7k}.$$

Find the smallest N, using The Estimation Theorem for Alternating Series, such that we can conclude

$$|S - S_N| < \frac{1}{10^4}.$$

A. 29999

B. 30000

C. 4284

D. 4285

E. 4286