MA 16600 Final Exam, Test number 26, December 2019

Name ______________________________

10–digit PUID number ______________________________

Recitation Instructor ______________________________

Recitation Section Number and Time ______________________________

Instructions: **MARK TEST NUMBER 26 ON YOUR SCANTRON**

1. Do not open this booklet until you are instructed to.
2. Fill in all the information requested above and on the scantron sheet.
3. This booklet contains 25 problems, each worth 8 points.
4. For each problem mark your answer on the scantron sheet and also **circle it in this booklet**.
5. Work only on the pages of this booklet.
6. Books, notes, calculators or any electronic device are not allowed during this test and they should not even be in sight in the exam room. You may not look at anybody else's test, and you may not communicate with anybody else, except, if you have a question, with your instructor.
7. You are not allowed to leave during the first 20 and the last 10 minutes of the exam.
8. When time is called at the end of the exam, put down your writing instruments and remain seated. The TAs will collect the scantrons and the booklets.
9. A collection of potentially useful identities:

$$\sin(a+b) = \sin a\cos b + \cos a \sin b \qquad 1-\cos 2a = 2\sin^2 a$$
$$\cos(a+b) = \cos a\cos b - \sin a \sin b \qquad 1+\cos 2a = 2\cos^2 a$$
$$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$$
$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

1. Find the length of the curve $y = f(x)$, $1 \le x \le \sqrt{5}$, given that the *derivative* of f is $f'(x) = \sqrt{x^3 - 1}$.

A. $\sqrt{5} - \dfrac{3}{4}$

B. $2(\sqrt[4]{5} - \dfrac{1}{5})$

C. $3\sqrt{5} - 2$

D. $2(\sqrt[3]{5} - 1)$

E. $3\sqrt{5} - \dfrac{2}{5}$

2. In the Taylor series of $\sqrt{x^3}$ about 2 the coefficient of $(x-2)^2$ is

A. $-\dfrac{3}{4}$

B. $\dfrac{\sqrt{3}}{4}$

C. $\dfrac{3}{8\sqrt{2}}$

D. $\dfrac{1}{2\sqrt{3}}$

E. $\dfrac{\sqrt{2}}{12}$

3. Given vectors $\boldsymbol{a} = 9\boldsymbol{i} - 3\boldsymbol{j} - 3\boldsymbol{k}$ and $\boldsymbol{b} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$, the projection of $\boldsymbol{a}$ on $\boldsymbol{b}$ is

A. $\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$

B. $4\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$

C. $\boldsymbol{i} + \boldsymbol{j} - 4\boldsymbol{k}$

D. $2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$

E. $2\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$

4. $\displaystyle\int_0^{\pi/2} \sin^{2/3} x \cos^3 x \, dx =$

A. 12/17

B. 1/2

C. 7/5

D. 8/25

E. 18/55

5. Find the volume generated when the region under the curve $y = \cos x$, $-\pi/2 \leq x \leq \pi/2$, is revolved about the y-axis.

A. $\pi^2 - 2\pi$

B. $4\pi/3$

C. $\pi^2/3$

D. $2 + \pi/2$

E. $2\pi - 1$

6. The interval of convergence of the series $\sum_{k=1}^{\infty} \frac{x^{k^2}}{k^k}$ is

A. $(-1, 1)$

B. $(-1, 1]$

C. $[-1, 1)$

D. $[-1, 1]$

E. None of the above.

7. A cylindrical boiler, mounted vertically, has radius $r = 1/2$ m and height $h = 4$ m. If it is full of water, how much work will it take to empty it by pumping the water over the top? Density of water is 10^3 kg/m^3, and gravitational acceleration is $g = 10$ m/s^2.

A. $2\pi 10^4$ J

B. $4\pi 10^4$ J

C. $8\pi 10^3$ J

D. $6\pi 10^3$ J

E. $5\pi 10^3$ J

8. A unit vector perpendicular to both $\langle 1, 0, 1\rangle$ and $\langle 1, 2, 0\rangle$ is

A. $\langle \frac{2}{5}, -\frac{1}{5}, -\frac{2}{5}\rangle$

B. $\langle \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\rangle$

C. $\langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\rangle$

D. $\langle -\frac{2}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\rangle$

E. $\langle \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\rangle$

9. The equation of the circle $(x-1)^2 + y^2 = 2$ in polar coordinates is

A. $r^2 - 2r\cos\theta = 1$

B. $-r\cos\theta + r\sin\theta = 2$

C. $r\cos\theta - r\sin\theta = 2$

D. $r^2 + 2r^2\sin\theta = 1$

E. $r^2 - 2r^3\cos\theta = -1$

10. $\displaystyle\int 3x^2 \sin^{-1} x\, dx =$

A. $x^3 \sin^{-1} x + \ln(1-x^2) + C$

B. $x^3 \cos^{-1} x + \ln\sqrt{1-x^2} + x^3 + C$

C. $x^2\sqrt{1-x^2} - 2x\sqrt{(1-x^2)^3} - \sin^{-1} x + C$

D. $x^3\sqrt{1-x^2} - 3(1-x^2)^{3/2} + \sin^{-1} x + C$

E. $x^3 \sin^{-1} x + \sqrt{1-x^2} - \frac{1}{3}(1-x^2)^{3/2} + C$

11. $\lim_{m\to\infty} \dfrac{\sqrt[3]{8m^3-1}}{\sqrt[4]{16m^4+24m^2}} =$

A. 0

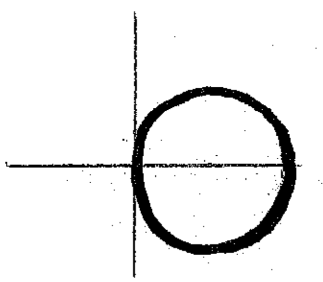
B. 3/2

C. 2

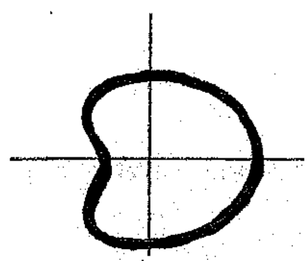
D. 1

E. 1/2

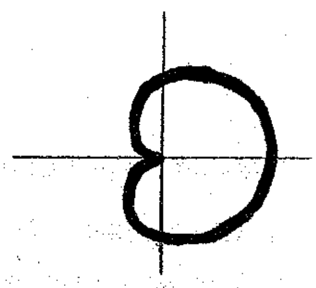
12. The polar equation $r = 1 + \sin 2\theta$ describes which of the following curves?



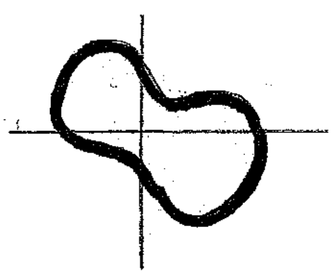
A.



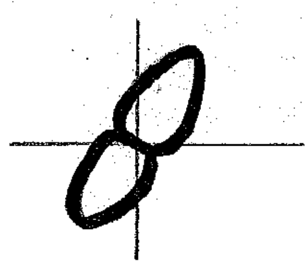
B.



C.



D.



E.

13. $\int_{-1}^{1} \frac{dx}{x^2-4} =$

A. $-\pi/2$

B. -2

C. $-\ln\sqrt{3}$

D. 0

E. $-2/3$

14. In 3 dimensional Cartesian coordinate system the equation $y = 3$ describes a

A. line parallel to the xz plane

B. line perpendicular to the y axis

C. line parallel to the y axis

D. plane perpendicular to the y axis

E. plane perpendicular to the xz plane

15. The Maclaurin series of $\dfrac{x+1}{x^2+1}$ is

A. $1 - x^2 + x^4 - x^6 + x^8 - \ldots$

B. $1 - x + x^2 - x^3 + x^4 - x^5 + \ldots$

C. $1 + x - x^2 - x^3 + x^4 + x^5 - \ldots$

D. $1 + x + x^2 + x^3 + x^4 + x^5 + \ldots$

E. $1 - x^2 - x^4 - x^6 - x^8 - \ldots$

16. $\displaystyle\sum_{m=1}^{\infty} \frac{3+2^m}{2^{2m}} =$

A. 1

B. 4/3

C. 5/2

D. 3/2

E. 2

17. To evaluate $\int \sqrt{1-4x-x^2}\,dx$, which substitution to make?

A. $x = 4\tan t - \dfrac{1}{2}$

B. $x = \sqrt{5}\sin t - 2$

C. $x = 3\sin t + \dfrac{1}{2}$

D. $x = \sqrt{2}\sec t + 3$

E. $x = 2\tan t - 3$

18. The base of a solid is a triangle in the xy plane, with vertices at $(0,0), (1,0), (0,1)$. If its cross sections perpendicular to the x axis are half discs with diameter on the xy plane, its volume is

A. $\pi/24$

B. π

C. $\pi/10$

D. $\pi/8$

E. $\pi/16$

19. What substitution should we make in order to evaluate $\int_0^5 (4+x^2)^{-1/2}\,dx$?

A. $x = \tan s$

B. $x = 2\tan s$

C. $x = 2\sec s$

D. $x = \sec s$

E. None of these substitutions will help.

20. The area between the curves given by polar equations $r = e^{\theta}$ and $r = 1$, $0 \le \theta \le 1$, is

A. $(e-1)/2$

B. $(e+3)/4$

C. $(e^3+e)/3$

D. $(e^2-3)/4$

E. $(e^3-4)/3$

21. The equation $x^2 + y^2 + z^2 - 2x - 4y + z = 0$ describes a sphere with center at

A. $(1, 2, -1/2)$
B. $(-1, 4, 2)$
C. $(-2, -4, 1)$
D. $(2, 4, 1)$
E. $(1/2, 1/4, -1)$

22. The Taylor series, centered at 0, of the function

$$g(x) = \begin{cases} \dfrac{\sin 2x}{x}, & \text{if } x \neq 0 \\ 2, & \text{if } x = 0 \end{cases}$$

is

A. $\displaystyle\sum_{k=0}^{\infty} 2^k \frac{x^k}{k!}$

B. $\displaystyle\sum_{k=0}^{\infty} (-2)^k \frac{x^{2k}}{(2k)!}$

C. $\displaystyle 2\sum_{k=0}^{\infty} (-4)^k \frac{x^{2k}}{(2k+1)!}$

D. $\displaystyle\sum_{k=0}^{\infty} 2^{2k+1} \frac{x^{2k+1}}{(2k)!}$

E. $\displaystyle 2\sum_{k=0}^{\infty} (-2)^{2k+1} \frac{x^{2k+1}}{(2k+1)!}$

23. Which is true?

I. If a series absolutely converges, then it converges;

II. If $\sum_{j=0}^{\infty} c_j$ converges, then $\lim_{j\to\infty} c_j = 0$;

III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converges if and only if $p > 0$.

A. Only I.

B. Only II.

C. Only I. and II.

D. Only II. and III.

E. All are true.

24. Which is true? The point whose Cartesian coordinates are $(3, \sqrt{3})$ has polar coordinates

I. $(2\sqrt{3}, \pi/6)$ II. $(-2\sqrt{3}, -\pi/6)$ III. $(2\sqrt{3}, -5\pi/6)$

A. Only I.

B. Only II.

C. Only I. and II.

D. Only I. and III.

E. All are true.

25. $\int_0^\infty xe^{-x^2}\,dx =$

A. 1/2

B. 2

C. 0

D. 1

E. The improper integral diverges.