SHOW YOUR WORK
SKETCH means DRAW CAREFULLY WITHOUT A DRAFTING KIT.

(1) Express $(a + b) \times (a - b)$ as a scalar multiple of $a \times b$.

(2) Recall that if $\mathbf{r}(t) = (x(t), y(t), z(t))$ is a space curve from $t = 0$ to $t = 1$, then the arc length $s(t)$ is given by

$$s(t) = \int_0^t \|\mathbf{r}'(\tau)\| \, d\tau.$$ 

Let

$$\mathbf{r}(t) = t\mathbf{i} + 2e^t \mathbf{j} + e^{2t} \mathbf{k}.$$ 

(a) Find $\mathbf{r}'(0)$. Find an equation for the line tangent to the curve at $(0, 2, 1)$.

(b) Find an equation for the plane normal to the curve at $(0, 2, 1)$.

(c) Find $\frac{d\mathbf{r}}{ds}(0)$. Find the unit tangent vector $\mathbf{T}(t)$.

(d) Find the curvature of the curve at $(0, 2, 1)$.

(e) What is the length of the curve between $(0, 2, 1)$ and $(1, 2e, e^2)$?

(3) A planet travels around the sun in an elliptical orbit of the form

$$\mathbf{r}(t) = 2 \cos t \mathbf{i} + 1.5 \sin t \mathbf{j}.$$ 

Find the velocity $\mathbf{v}(t)$ and the acceleration $\mathbf{a}(t)$. What is the maximum magnitude of the acceleration?

(4) Sketch in the $x$–$y$ plane the level curve $f(x, y) = 0$ when

$$f(x, y) = \ln(x^2 y^2).$$ 

(5) Sketch the surface

$$x^2 - y^2 + z^2 + 1 = 0.$$