(10 points) Find the directional derivative of the function \( F(x, y, z) = e^x \sin(yz) \) at the point \((0, 1, \pi)\) in the direction of most rapid increase.

(15 points)
(a) Write an equation for the line passing through the points \((1, 2, 3)\) and \((-1, 1, 0)\).
(b) Write an equation for the plane containing the points \((1, -1, 2), (2, 1, 1), \) and \((0, 1, 2)\).
(c) Find the angle between the planes given by the equations \(2x - y = -2\) and \(x - 2z = 1\).
(10 points) Let \( \mathbf{r}(t) = (e^t \sin t, e^t \cos t) \) for \( 0 \leq t \leq 2 \). What is the length of this curve?

(15 points) Consider the graph \( y = \ln x \) for \( x > 0 \). Recall that the curvature \( k(x) \) of the curve \( y = f(x) \) is given by the formula

\[
k(x) = \frac{|y''(x)|}{[1 + (y'(x))^2]^{3/2}}.
\]

(a) Show that \( k(x) = x/(1 + x^2)^{3/2} \).

(b) Find the point on the curve for which the curvature is maximal. What is the curvature there?
(10 points) Find the tangent plane to the surface given by the formula

\[ x^2 y^2 + e^{yz} = 2 \]

at the point \((1, 1, 0)\).

(15 points) Show that for any two vectors \(a\) and \(b\),

\[ \|a + b\|^2 - \|a - b\|^2 = 4 \, a \cdot b. \]

(Hint: \(\|v\|^2 = v \cdot v\).)
(20 points) Find the critical points of the function \( f(x, y) = (x^3 - 3x)(1 + y^2) \) and classify each critical point as a maximum, minimum, or saddle point.

(15 points) Let \( U \) be the sphere \( x^2 + y^2 + z^2 \leq 1 \), let \( S \) be the boundary surface of \( U \), and let \( \mathbf{n} \) be the outward normal to \( S \). Let \( \mathbf{v}(x, y, z) = x \mathbf{i} + y \mathbf{j} + z \mathbf{k} \). Evaluate \( \int \int_S \mathbf{v} \cdot \mathbf{n} \, d\sigma \).
(10 points) Let $T$ be the region bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + 2y + 3z = 6$.
(a) Carefully sketch the region $T$ below.
(b) Fill in the boxes; do not evaluate the integral.

Volume of $T = \int \int \int dx \, dy \, dz$.

(20 points) Find the centroid of the solid bounded above by the surface $z - (x^2 + y^2) = 0$, below by the plane $z = 0$, and at the sides by the cylinder $x^2 + y^2 = 1$. Is the centroid inside the solid? Justify your answer. (Hint: Sketching the solid may help reduce the computation.)
(15 points) An object travels along the ellipse \( x = a \cos t, \ y = b \sin t \) for \( 0 \leq t \leq 2\pi \) subject to a force \( \mathbf{F}(x, y) = -\frac{1}{2}[y \mathbf{i} - x \mathbf{j}] \).

(a) Write the work done by the force as a line integral.

(b) Express the work done by the force in terms of the area of the ellipse.

(20 points) Calculate the line integral of the vector field

\[
\mathbf{F}(x, y) = \frac{x}{(x^2 + y^3)^{1/2}} \mathbf{i} + \frac{3y^2}{2(x^2 + y^3)^{1/2}} \mathbf{j}
\]

along the circular arc \( x^2 + y^2 = 2 \) from the point \((1, 1)\) counter-clockwise to the point \((0, \sqrt{2})\).
Let the surface $S$ be the upper hemisphere given by the formula $x^2 + y^2 + z^2 = 4$ for $z \geq 0$. Let $\mathbf{n}$ be the upper unit normal of $S$, and let the curve $C$ be the boundary of $S$. Let $\mathbf{v}(x, y, z) = -y \mathbf{i} + x \mathbf{j}$. Calculate

$$\iint_S \text{curl} \mathbf{v} \cdot \mathbf{n} \, d\sigma$$

(a) as a surface integral.
(b) as a line integral using Stokes’ theorem.