

- (12 pts) 1. Find all relative maxima, minima, and saddle points for

$$f(x, y) = 2xy - x^2 - y^2 + 3x + 4.$$

- (14 pts) 2. (a) If  $f(x, y, z) = z - x^2 - y^2$  find  $\nabla f$ .

(b) Find the tangent plane of  $z - x^2 - y^2 = 4$  at  $(1, 1, 6)$ .

(c) Find equations for the normal line to  $z - x^2 - y^2 = 4$  at  $(1, 1, 6)$ .

- (12 pts) 3. If

$$w = x^2 + y^2 + z^2,$$

$$x = u^2 - v^2, \text{ and}$$

$$y = u^2 + v^2$$

find  $\frac{\partial w}{\partial u}$  at  $(u, v) = (1, -1)$ .

- (14 pts) 4. a) If  $f(x, y, z) = x^3 - xy^2 - z$  find the directional derivative of  $f$  at  $(1, 1, 0)$  in the direction of  $\vec{v} = \frac{i}{\sqrt{2}} + \frac{j}{\sqrt{2}}$ .

b) In what direction does  $f$  change most rapidly at  $(1, 1, 0)$ ?

c) What is the approximate change in  $f$  as you move from  $(1, 1, 0)$  to  $(3, 3, 0)$ ?

- (10 pts) 5. If  $x^2 - xy + y^2 + zw - w^2 = 5$  subject to  $x + y - t = 0$  find  $\left(\frac{\partial w}{\partial x}\right)_t$ , in other words  $x$  and  $t$  are independent, at  $(x, y, z, w) = (2, 1, 3, 1)$ .

- (14 pts) 6. If  $f(x, y) = \sin x \cos y + x^2 + y^2$  find

a) the linearization of  $f$  at  $(0, 0)$ ,

b) the quadratic approximation of  $f$  at  $(0, 0)$ .

c) Estimate the maximum value of  $|f(x, y) - \ell(x, y)|$  if  $|x| < \frac{1}{100}$  and  $|y| < \frac{1}{100}$ .

- (12 pts) 7. Let  $f(x, y) = x^2 + 2y^2 - x$  on the set  $\{(x, y) | x^2 + y^2 \leq 1\}$  find the absolute maximum value of the function.

- (12 pts) 8. Find the points on the surface  $4xy - z - \frac{9}{4} = 0$  which are closest to the origin.