MATHEMATICS 182 FINAL EXAM

(20 pts) 1) Let \( f(x, y, z) = x^2 \cos y + z^2. \)
   a) Find \( \nabla f \) at \((1, 0, 1)\).
   b) Find the directional derivative of \( f \) in the direction of \( \vec{V} = 2k \).
   c) In what direction does \( f \) change most rapidly?

(20 pts) 2) Given the points \( P_1 = (1, 2, 1), \ P_2(2,2,3), \) and \( P_3 = (2, -1, 1) \) find
   a) the equations of a line through \( P_1 \) and \( P_2 \),
   b) the equation of a plane through \( P_1, P_2, \) and \( P_3 \).

(15 pts) 3) If \( w = x^2 + y^2, \ x = r - s, \) and \( y = r + s \) find \( \frac{\partial^2 w}{\partial r^2} + \frac{\partial^2 w}{\partial s^2} \)

(25 pts) 4) a) Show that the vector field
    \[ \vec{F} = (z + \cos y)i + [-x \sin y]j + (x + z^2)k \]
    is conservative.
   b) Find \( f \) such that \( \vec{F} = \nabla f \).
   c) What is \( \int_C \vec{F} \cdot d\vec{R} \) if \( C \) is the curve \( x = t^3, \ y = \pi t^2, \ z = t, \ 0 \leq t \leq 1. \)

(20 pts) 5) a) Find parametric equations for the cone
    \[ 3(x^2 + y^2) = z^2, \ 0 \leq z \leq \sqrt{3} \]
   b) Find the surface area of the cone.
(20 pts) 6) a) Use the divergence theorem to evaluate \[ \int_S (\nabla \cdot \vec{V}) \, d\sigma \] where \( \vec{V} = 3xi - yj - zk \) and \( S \) is the sphere \( x^2 + y^2 + z^2 = 9 \).

b) Use Stoke's Theorem to evaluate \[ \int_S (\nabla \times \vec{V}) \cdot d\vec{N} \, d\sigma \] if \( \vec{V} = x^2y^2i \) and \( S \) is the upper half of \( x^2 + y^2 + z^2 = 9 \).

(20 pts) 7) Find the mass of the paraboloid \( z = 1 - x^2 - y^2, \ z \geq 0 \), if the density \( \int = x^2 \).

(25 pts) 8) If \( x = \cos t, \ y = \sin t, \) and \( z = 4t \) find

a) the unit tangent vector as a function of \( t \),

b) \( \frac{ds}{dt} \) as a function of \( t \),

c) the curvature \( \kappa = \frac{|dT|}{ds} \) as a function of \( t \).

(20 pts) 9) a) Find all critical points of
\[ f(x, y) = 9x^3 + y^3/3 - 4xy \]

b) Decide which of the critical points are maxima, minima, at saddle points.

(15 pts) 10) a) If a twice differentiable vector field \( \vec{V} = v, i + v_2j + v_3k \) is the gradient of a scalar function \( f \), \( \vec{V} = \nabla f \), show \( \nabla \times \vec{V} = 0 \).

b) If \( \int_C \vec{V} \cdot d\vec{R} \) depends on the endpoints only and if \( \vec{V} \) is twice differentiable show \( \vec{V} = \nabla f \) for some scalar function \( f \).