INSTRUCTIONS

1. Fill in your name and your instructor's name above.
2. You must use a \#2 pencil on the scantron answer sheet.
3. Fill in your name, your four digit section number, and your student identification number. Make sure to blacken in the appropriate spaces. If you do not know your section number, ask your instructor. (Leave the test/quiz number blank.) Sign your name.
4. There are 15 questions. Blacken in your choice of the correct answer in the spaces provided on the scantron answer sheet. Only the scantron answer sheet will be graded. When you have completed the exam, turn in the scantron answer sheet only. You may take the exam booklet with you.
5. The exam is self-explanatory. Do not ask your instructor any questions about the exam problems.
6. Only one-line calculators (any brand) are allowed. Cell phones and PDA’s may not be used as a calculator and must be put away during the exam. NO BOOKS OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

CONSUMERS’ AND PRODUCERS’ SURPLUS

\[
CS = \int_0^{q_0} D(q) dq - p_0 q_0 \\
PS = p_0 q_0 - \int_0^{q_0} S(q) dq
\]

VOLUME & SURFACE AREA

<table>
<thead>
<tr>
<th>Right Circular Cylinder</th>
<th>Sphere</th>
<th>Right Circular Cone</th>
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</thead>
<tbody>
<tr>
<td>( V = \pi r^2 h )</td>
<td>( V = \frac{4}{3} \pi r^3 )</td>
<td>( V = \frac{1}{3} \pi r^2 h )</td>
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<tr>
<td>( SA = \left{ \begin{array}{l} 2\pi r^2 + 2\pi rh \ \pi r^2 + 2\pi rh \end{array} \right. )</td>
<td>( SA = 4\pi r^2 )</td>
<td>( SA = \pi r \sqrt{r^2 + h^2} + \pi r^2 )</td>
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1. A manufacturer estimates that the marginal profit of producing \( q \) units of a certain commodity is \( P'(q) = -2q + 105 \). If the profit derived from producing 10 units is $450, how much profit should be expected from producing 30 units?

A. $300  
B. $1350  
C. $2255  
D. $500  
E. $1750

2. Evaluate the indefinite integral.  
\[
\int x^2e^{x^3} \, dx
\]

A. \( \frac{1}{3} x^3e^{x^3+1} + C \)  
B. \( 3e^{x^3} + C \)  
C. \( 3x^2e^{x^3} + C \)  
D. \( \frac{1}{3} x^3e^{x^3} + C \)  
E. \( \frac{1}{3} e^{x^3} + C \)

3. Use the Fundamental Theorem of Calculus to evaluate the definite integral.  
\[
\int_{-2}^{1} t^2 \, dt
\]

A. 9  
B. 3  
C. \(-7/3\)  
D. 5  
E. \(-3\)
4. The area of the triangle bounded by the lines $y = 2x$, $x = 2$, and the $x$ axis is:

A. 1
B. 2
C. 3
D. 4
E. 5

5. Find the average value of the following function over the interval $2 \leq x \leq 4$

$$f(x) = \frac{1}{x^2} - \frac{1}{x^3}$$

A. $\frac{64}{3}$
B. $\frac{5}{64}$
C. $\frac{128}{3}$
D. $\frac{5}{32}$
E. $\frac{3}{2\sqrt{4}} - \frac{3\sqrt{4}}{2} - \frac{2\sqrt{2}}{3} + \frac{8}{3}$
6. The slope at each point \((x, y)\) on a curve \(y = f(x)\) is given by:

\[ f'(x) = e^x (e^{-x} - 1). \]

Find \(f(x)\) given that the point \((0, 1)\) lies on the curve.

A. \(f(x) = x - e^x + 2\)
B. \(f(x) = 1 - xe^x\)
C. \(f(x) = e^x (-e^{-x} - x) + 2\)
D. \(f(x) = \frac{1}{2} (e^{-x} - 1)^2 + 1\)
E. \(f(x) = \frac{1}{2} e^{2x} - e^x + 1\)

7. Use the rules for definite integrals and that

\[
\begin{align*}
\int_{-3}^{2} f(x) \, dx &= 5 \\
\int_{-3}^{2} g(x) \, dx &= -2 \\
\int_{-3}^{4} f(x) \, dx &= 7 \\
\int_{-3}^{4} g(x) \, dx &= 1
\end{align*}
\]

to compute

\[
\int_{-3}^{2} [3f(x) - 4g(x)] \, dx
\]

A. 3
B. 14
C. 23
D. 7
E. 26
8. The slope $f'(x)$ at each point $(x, y)$ on a curve $y = f(x)$ is given, along with a point $(a, b)$ on the curve. Use this information to find $f(x)$.

$$f'(x) = \frac{4x}{1 + 4x^2}; \quad (2, 0)$$

A. $f(x) = \frac{1}{2} \ln(|1 + 4x^2|)$

B. $f(x) = \frac{1}{2} \ln(17(|1 + 4x^2|))$

C. $f(x) = \frac{1}{2} \ln(289(|1 + 4x^2|))$

D. $f(x) = \frac{1}{2} \ln \left( \frac{|1 + 4x^2|}{17} \right)$

E. $f(x) = \frac{1}{2} \ln \left( \frac{|1 + 4x^2|}{289} \right)$

9. At $t$ hours past midnight, the temperature $T$ (in degrees Celsius) in West Lafayette is found to be changing at a rate given by

$$T'(t) = 0.03(t^2 - 8t) \text{ degrees/hour}.$$

By how many degrees will the temperature change from midnight to 10 AM?

A. $-2$

B. $-1$

C. 0

D. 1

E. 2
10. A supplier of hair dryers will make \( q \) hundred units of hair dryers available in the market according to a supply function \( S(q) = \sqrt{36 + 1.8q} \) dollars per unit. Determine the producer’s surplus if the price is set at $9 per unit.

A. $1,800  
B. $530  
C. $4,500  
D. $11,700  
E. $3,500

11. Find the indicated integral:

\[
\int t^{1/2} \left( t + \frac{1}{\sqrt{t^3}} + \frac{e^{5t}}{\sqrt{t}} \right) dt.
\]

A. \( \frac{2}{5} t^{5/2} + \ln |t| + \frac{1}{5} e^{5t} + C \)  
B. \( \frac{2}{3} t^{3/2} + 4t^{1/4} + \frac{1}{5} e^t + C \)  
C. \( \frac{2}{3} t^{3/2} + \frac{1}{4} t^{1/4} + \frac{1}{5} e^{5t} + C \)  
D. \( \frac{2}{5} t^{5/2} + \ln |t| + \frac{1}{5} e^t + C \)  
E. \( \frac{2}{3} t^{3/2} + 4t^{1/4} + 5e^{5t} + C \)
12. A biologist determines that, $t$ hours after a bacterial colony was established, the
population of bacteria in the colony is changing at a rate given by

$$P'(t) = \frac{2e^t}{1 + e^t}$$

million bacteria per hour, $0 \leq t \leq 8$.

If the bacterial colony started with a population of 1 million, how many bacteria are
present in the colony after the 8-hour experiment? Round your answer to 2 decimal
places

A. 15.61 million
B. 14.61 million
C. 16.39 million
D. 15.00 million
E. 17.00 million

13. Find the area of the region under the curve of $f(x) = x(x - 2)^3$ over the interval
$2 \leq x \leq 3$.

A. $\frac{65}{4}$
B. $\frac{747}{10}$
C. $\frac{9}{20}$
D. $\frac{7}{10}$
E. $\frac{1}{4}$
14. Find the area of the region bounded by the curves \( y = x^3 + x^2 \) and \( y = 6x \). Round your answer to 2 decimal places.

A. 5.33
B. 15.75
C. 21.08
D. 10.42
E. 26.17

15. When the brake is applied to a vehicle, the speed of the vehicle is decreased at the rate of \( v'(t) = -2t + 1 \). Suppose the speed of the vehicle is 12 mi/h before the brake is applied, find the average speed of the vehicle over the time period between when the break is applied and the vehicle stops.

A. 0.5
B. 2.85
C. 5.32
D. 8.67
E. 10.16