1. You must use a #2 pencil on the scantron answer sheet.
2. Fill in your name, your four digit section number, and your student identification number. If you do not know your section number, ask your instructor. (Leave the test/quiz number blank.) Sign your name.
3. There are 15 questions. Blacken in your choice of the correct answer in the spaces provided on the scantron answer sheet. **Only the scantron answer sheet will be graded**.
4. The exam is self-explanatory. Do not ask your instructor any questions about the exam problems.
5. Only one-line calculators (any brand) are allowed. Cell phones and PDA's may not be used as a calculator and must be put away during the exam. NO BOOKS OR PAPERS ARE ALLOWED.

CONSUMERS' AND PRODUCERS' SURPLUS

$$\text{CS} = \int_0^{q_0} D(q)dq - p_0q_0 \qquad \text{PS} = p_0q_0 - \int_0^{q_0} S(q)dq$$

TRAPEZOIDAL RULE

$$\int_a^b f(x)dx \equiv \frac{\Delta x}{2}\Big[f(x_1) + 2f(x_2) + 2f(x_3) + \cdots + 2f(x_n) + f(x_{n+1})\Big],$$

where $a = x_1, x_2, x_3, \ldots, x_{n+1} = b$ subdivides $[a, b]$ into n equal subintervals of length $\Delta x = \dfrac{b-a}{n}$.

THE SECOND DERIVATIVE TEST

Suppose f is a function of two variables x and y, and that all the second-order partial derivatives are continuous. Let

$$D = f_{xx}f_{yy} - (f_{xy})^2$$

and suppose (a, b) is a critical point of f.

1. If $D(a, b) < 0$, then f has a saddle point at (a, b).
2. If $D(a, b) > 0$ and $f_{xx}(a, b) < 0$, then f has a relative maximum at (a, b).
3. If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then f has a relative minimum at (a, b).
4. If $D(a, b) = 0$, the test is inconclusive.

LAGRANGE EQUATIONS

For the function $f(x, y)$ subject to the constraint $g(x, y) = k$, the Lagrange equations are

$$f_x = \lambda g_x \qquad f_y = \lambda g_y \qquad g(x, y) = k$$

LEAST-SQUARES LINE

The equation of the least-squares line for the n points (x_1,y_1), (x_2,y_2), ... , (x_n,y_n), is $y = mx + b$, where

$$m = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2} \qquad b = \frac{\sum x^2 \sum y - \sum x \sum xy}{n\sum x^2 - \left(\sum x\right)^2}$$

VOLUME & SURFACE AREA

Right Circular Cylinder

$V = \pi r^2 h$

$$SA = \begin{cases} 2\pi r^2 + 2\pi rh \\ \pi r^2 + 2\pi rh \end{cases}$$

Sphere

$V = \frac{4}{3}\pi r^3$

$SA = 4\pi r^2$

Right Circular Cone

$V = \frac{1}{3}\pi r^2 h$

$SA = \pi r\sqrt{r^2 + h^2} + \pi r^2$

1. Use the trapezoidal rule to approximate the given integral.

$$\int_3^5 \frac{x}{\ln x}\, dx; \;\; n = 4$$

A. 5.79

B. 3.63

C. 11.58

D. 4.41

E. 7.25

2. Suppose $z = \ln(5x^2 + 11xy - 3y^2)$, where $x = \dfrac{1}{\sqrt{t}}$, and $y = \sqrt{t}$, find $\dfrac{dz}{dt}$ in terms of x, y, and t.

A. $\dfrac{1}{5x^2 + 11xy - 3y^2}\left(\dfrac{11x - 6y}{\sqrt{t}} + \dfrac{10x + 11y}{t^{3/2}}\right)$

B. $\dfrac{1}{2(5x^2 + 11xy - 3y^2)}\left(\dfrac{1}{\sqrt{t}} - \dfrac{1}{t^{3/2}}\right)$

C. $\dfrac{1}{2(5x^2 + 11xy - 3y^2)}\left(\dfrac{11x - 6y}{\sqrt{t}} - \dfrac{10x + 11y}{t^{3/2}}\right)$

D. $\dfrac{1}{5x^2 + 11xy - 3y^2}\left(\dfrac{1}{\sqrt{t}} + \dfrac{1}{t^{3/2}}\right)$

E. $\dfrac{1}{2(5x^2 + 11xy - 3y^2)}\left(\dfrac{11x - 6y}{\sqrt{t}} + \dfrac{10x + 11y}{t^{3/2}}\right)$

3. Below is the data for the prices of a certain product since it became available in the market back in 2009.

Year	2009	2010	2011	2012
Price	\$200	\$192	\$180	\$170

Using the method of least-squares, predict the price of the product in 2015.

A. \$138.8

B. \$139.6

C. \$140

D. \$149.8

E. \$150

4. A dealer determines that if a certain type of vehicle is sold for x dollars apiece and the price of the gasoline for this vehicle is y dollars per gallon, then approximately H units will be sold each day, where $H(x, y) = 13 - 19x^{\frac{1}{2}} + 6y^{3/2}$. The current price of the vehicle is 40000 dollars apiece, and the current price of the gasoline is 4 dollars per gallon. Use calculus to estimate the change in demand for this vehicle if the price of the vehicle is reduced by 400 dollars apiece and the price of gasoline increases by 1 dollar per gallon.

A. Increase by 30 units

B. Increase by 34 units

C. Decrease by 30 units

D. Decrease by 1 unit

E. Increase by 37 units

5. Evaluate

$$\int_2^{+\infty} x^2(x^3-1)^{-2}\,dx$$

A. $1/7$

B. $4/49$

C. $1/6$

D. It diverges.

E. $1/21$

6. For the function $z = \dfrac{xy^2}{x^2y^3+1}$, the first partial derivative $\dfrac{\partial z}{\partial y}$ is

A. $\dfrac{2xy - x^3y^4}{x^2y^3+1}$

B. $\dfrac{2xy - x^3y^4 - xy^2}{(x^2y^3+1)^2}$

C. $\dfrac{2xy + x^3y^4}{(x^2y^3+1)^2}$

D. $\dfrac{y^2 - x^2y^5}{(x^2y^3+1)^2}$

E. $\dfrac{2xy - x^3y^4}{(x^2y^3+1)^2}$

7. The domain of the function $f(x,y) = \dfrac{\ln(9-x-y)}{\sqrt[4]{x+y-4}}\sqrt{x+y-2}$ is all ordered pairs (x,y) of real numbers such that:

A. $4 < x+y < 9$

B. $2 < x+y \le 9$

C. $4 \le x+y < 9$

D. $4 \le x+y \le 9$

E. $2 \le x+y < 9$

8. Find the following indefinite integral.

$$\int (x+1)e^{2x}\,dx$$

A. $\dfrac{(x^2+x)e^{2x}}{4}+C$

B. $2e^{2x}+C$

C. $\dfrac{(x^2+x)e^{2x+1}}{2}+C$

D. $\dfrac{(2x+1)e^{2x}}{4}+C$

E. $\dfrac{(x+1)e^{2x}}{2}+C$

9. Given $f(x,y)=x^2-y^2+2x$ and $g(x,y)=x^2+y^2$. In the process of finding the extremum value(s) of $f(x,y)$ subject to the constraint $g(x,y)=1$, which of the following statements are true?

I. There are 2 points at which the constrained minimum value is achieved.

II. There are 4 points at which the constrained extrema are achieved.

III. The constrained maximum value of $f(x,y)$ is 3 at $(1,0)$ and the constrained minimum of value $f(x,y)$ is -1 at $(-1,0)$.

A. I, II, and III are true.

B. Only I and II are true.

C. Only I and III are true.

D. Only I is true.

E. Only II is true.

10. You are driving in a desert and you meet a starving alien. You give him food, and in return he scans the whole desert. He tells you that the probability that you will find a good place to dig a new oil well x miles east, y miles north of the current location is $P(x, y) = \frac{1}{e^{x^2+y^2+2x+1}}$, and he leaves. Where should you try to dig a new oil well?

   A. Current location

   B. 1 mile north of the current location

   C. 1 mile east of the current location

   D. 1 mile west of the current location

   E. 1 mile south of the current location

11. Find the definite integral. Approximate the answer to the nearest hundredth.

$$\int_1^{e^2} (t^2 + t + 1) \ln \sqrt{t}\, dt$$

   A. 25.46

   B. 136.91

   C. 28.46

   D. 62.99

   E. 213.32

12. Assuming a constant interest rate, it is estimated that after t years, the value $V(t)$ of a certain business will be changing at a rate of

$$V'(t) = (2.1t - 1.5)e^{-0.05t} \qquad \text{million dollars per year.}$$

To the nearest million dollars, what is the net change of the value over the next 4 years?

A. 9 million dollars

B. 8 million dollars

C. 7 million dollars

D. 11 million dollars

E. 5 million dollars

13. The demand for a product is

$$Q(x, y) = 200 - 10x^2 + 20xy \text{ units per month,}$$

where x is the price of the product and y is the price of a competing product. It is estimated that t months from now, the price of the product will be

$$x(t) = 10 + 0.5t \text{ dollars per unit}$$

and the price of the competing product will be

$$y(t) = 12.8 + 0.2t^2 \text{ dollars per unit.}$$

At what rate will the demand for the product be changing with respect to time 4 months from now?

A. 424 units per month/month

B. 128 units per month/month

C. 324 units per month/month

D. 80 units per month/month

E. 320 units per month/month

14. The function $f(x,y) = x^2e^y - xy$ has 2 critical points: $(0,0)$, and $\left(e^{-2},2\right)$. Select the correct classification for those critical points:

A. $(0,0)$ is saddle point and $\left(e^{-2},2\right)$ is a relative maximum.

B. $(0,0)$ is saddle point and $\left(e^{-2},2\right)$ is a relative minimum.

C. Both $(0,0)$ and $\left(e^{-2},2\right)$ are saddle points.

D. $\left(e^{-2},2\right)$ is saddle point and $(0,0)$ is a relative maximum.

E. $\left(e^{-2},2\right)$ is saddle point and $(0,0)$ is a relative minimum.

15. An aquarium, a rectangular box without a top, will be made of 2 different kinds of glass. The glass for the bottom costs \$3 per ft^2, and the glass for the sides costs \$2 per ft^2.

If the aquarium must have a volume of 48 ft^3, find the *height* of the aquarium so that the cost of the aquarium is a minimum.

A. 3 ft.

B. 4 ft.

C. 6 ft.

D. 8 ft.

E. 12 ft.