1. You must use a <u>#2 pencil</u> on the scantron answer sheet.
2. Fill in your <u>name</u>, your four digit <u>section number</u>, "01" for the <u>Test/Quiz Number</u>, and your <u>student identification number</u>. Make sure to blacken in the appropriate spaces. If you do not know your section number, ask your instructor. <u>Sign your name.</u>
3. There are 15 questions. Blacken in your choice of the correct answer on the scantron answer sheet. **Only the scantron answer sheet will be graded. Turn in the scantron answer sheet only. You may take the exam booklet with you.**
4. **The exam is self-explanatory. <u>Do not</u> ask your instructor any questions about the exam problems.**
5. **Only one-line calculators are allowed. Cell phones and PDA's may not be used during the exam and must be put away. NO BOOKS OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.**

CONSUMERS' AND PRODUCERS' SURPLUS

$$\mathbf{CS} = \int_0^{q_0} D(q)dq - p_0q_0 \qquad \mathbf{PS} = p_0q_0 - \int_0^{q_0} S(q)dq$$

TRAPEZOIDAL RULE

$$\int_a^b f(x)dx \equiv \frac{\Delta x}{2}\left[f(x_1) + 2f(x_2) + 2f(x_3) + \cdots + 2f(x_n) + f(x_{n+1})\right],$$

where $a = x_1, x_2, x_3, \ldots, x_{n+1} = b$ **subdivides** $[a, b]$ **into** n **equal subintervals of length** $\Delta x = \dfrac{b-a}{n}$**.**

THE SECOND DERIVATIVE TEST

Suppose f **is a function of two variables** x **and** y**, and that all the second-order partial derivatives are continuous. Let**

$$D = f_{xx}f_{yy} - (f_{xy})^2$$

and suppose (a, b) **is a critical point of** f**.**

1. **If** $D(a, b) < 0$**, then** f **has a saddle point at** (a, b)**.**
2. **If** $D(a, b) > 0$ **and** $f_{xx}(a, b) < 0$**, then** f **has a relative maximum at** (a, b)**.**
3. **If** $D(a, b) > 0$ **and** $f_{xx}(a, b) > 0$**, then** f **has a relative minimum at** (a, b)**.**
4. **If** $D(a, b) = 0$**, the test is inconclusive.**

LEAST-SQUARES LINE

The equation of the least-squares line for the n **points** (x_1,y_1), (x_2,y_2), ... , (x_n,y_n), **is**
$y = mx + b$, **where**

$$m = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2} \qquad b = \frac{\sum x^2 \sum y - \sum x \sum xy}{n\sum x^2 - \left(\sum x\right)^2}$$

LAGRANGE EQUATIONS

For the function $f(x, y)$ **subject to the constraint** $g(x, y) = k$**, the Lagrange equations are**

$$f_x = \lambda g_x \qquad f_y = \lambda g_y \qquad g(x, y) = k$$

VOLUME & SURFACE AREA

Right Circular Cylinder	Sphere	Right Circular Cone
$V = \pi r^2 h$	$V = \frac{4}{3}\pi r^3$	$V = \frac{1}{3}\pi r^2 h$
$SA = \begin{cases} 2\pi r^2 + 2\pi rh \\ \pi r^2 + 2\pi rh \end{cases}$	$SA = 4\pi r^2$	$SA = \pi r\sqrt{r^2 + h^2} + \pi r^2$

1. Use Integration by Parts to evaluate $\int (2x+1)\ln x dx$.

   A. $(x^2+x)\ln x - \frac{x^2}{2} - x + C$

   B. $\ln x[\frac{1}{2}\ x^2 + x - \frac{1}{3}\ x^3] + C$

   C. $\dfrac{2x+1}{x} - 2\ln|x| + C$

   D. $(2x+1)\ \frac{(\ln x)^2}{2} - \frac{1}{3}(\ln x)^3 + C$

   E. $\frac{(2x+1)^2}{4x} + C$

2. What do the level curves for $f(x,y) = \sqrt{y+x^2}$ look like?

   A. Point at the origin

   B. Lines

   C. Circles

   D. Parabolas

   E. Hyperbolas

3. The area A enclosed by an ellipse is $A = \pi ab$ where a is the length of the semi-major axis and b is the length of the semi-minor axis. Consider an ellipse with semi-major axis of length 50 inches and semi-minor axis of length 30 inches. If currently the semi-major axis is getting longer at the rate of 4 inches per second and the semi-minor axis is getting shorter at the rate of 2 inches per second, then at what rate is the area A changing?

   A. 8π square inches per second

   B. 12π square inches per second

   C. 20π square inches per second

   D. 140π square inches per second

   E. 260π square inches per second

4. The cash value of a company is estimated on January 1 of several consecutive years, and the results are summarized as follows:

Year	2010	2011	2012	2013	2014
Value (million \$)	2	4	6	7	8

Use the least-squares line to estimate the cash value (in millions of dollars) of the company on January 1, 2015.

A. 8.5

B. 9

C. 9.2

D. 9.5

E. 9.9

5. Records indicate that t hours after 6:00 AM the temperature at the Purdue Airport was $f(t) = t\sqrt{t^2 + 1}$ degrees Celsius. What was the average temperature at the airport between 8:00 AM and noon?

A. $71.2^o C$

B. $20.5^o C$

C. $17.8^o C$

D. $101.8^o C$

E. $18.4^o C$

6. Approximate the given integral using the trapezoid rule with the specified number of sub-intervals.
$$\int_1^4 \frac{\ln(x)}{e^x}\,dx\ ;\ n = 6$$

A. 0.16655

B. 0.36471

C. 0.18236

D. 0.09435

E. 0.19053

7. If $f(x, y) = e^{x^2+5y} + \dfrac{x}{y} - 2y^3$, then what is f_{xy}?

A. $10xe^{x^2+5y} - \dfrac{1}{y^2}$

B. $10xe^{x^2+5y} - \dfrac{1}{y^2} - 6y^2$

C. $e^{x^2+5y} - \dfrac{1}{y^2}$

D. $e^{x^2+5y} - \dfrac{1}{y^2} - 6y^2$

E. $10xye^{x^2+5y} - \dfrac{1}{y^2}$

8. The daily output of a factory is given by

$$Q(x,y) = 100\ln(x^2 + 10xy + 1)$$

million units, where x is the number of worker-hours each day and y the amount of power used each day in megawatt-hours. Currently, 50 worker-hours and 4 megawatt-hours are used each day. Use increments to approximate the increase in the daily output of the factory (in millions of units) if we increase the worker-hours by 2 and decrease the power used by 0.1 megawatt-hour. Round to 2 decimal places.

A. 0.07

B. 5.03

C. 5.11

D. 7.33

E. 8.41

9. Find all critical points of $f(x,y) = xy^2 + 8x^2 + 4y^2$ and classify each as a relative maximum, a relative minimum, or a saddle point.

A. relative maximum: $(0,0)$, saddle point: $(-4,8)$

B. relative maximum: $(0,0)$, saddle point: $(-4,-8),(-4,8)$

C. relative minimum: $(0,0)$, saddle point: $(-4,8)$

D. relative minimum: $(0,0)$, saddle point: $(-4,-8),(-4,8)$

E. relative minimum: $(0,0),(-4,0)$, saddle point: $(-4,8)$

10. Solve the following initial value problem using integration by parts: $\begin{cases} \dfrac{dy}{dx} = (3x+1)\sqrt{2x-3} \\ y = 0 \text{ when } x = 2 \end{cases}$

A. y=$\dfrac{(3x+1)(2x-3)^{3/2}}{3} - \dfrac{(2x-3)^{5/2}}{5} - \dfrac{32}{15}$

B. y=$(3x+1)(2x-3)^{3/2} - \dfrac{3(2x-3)^{5/2}}{5} - \dfrac{32}{5}$

C. y=$\dfrac{(3x+1)(2x-3)^{-1/2}}{2} - \dfrac{2(2x-3)^{-3/2}}{3} - \dfrac{5}{2}$

D. y=$\dfrac{(3x+1)^2(2x-3)^{1/2}}{6} - \dfrac{2(2x-3)^{-1/2}}{3} - \dfrac{1}{2}$

E. y=$\dfrac{(3x+1)^{1/2}(2x-3)}{2} - \dfrac{3(2x-3)^2}{2} - 2$

11. If the demand and supply functions for a certain commodity are $D(q) = -0.1q^2 + 100$ dollars per unit and $S(q) = 0.2q^2 + q + 60$ dollars per unit. Determine the consumer surplus at the equilibrium price.

A. \$34.29

B. \$16,200.00

C. \$183.33

D. \$89.36

E. \$66.67

12. Compute $\int_0^\infty \frac{x}{e^x} dx$.

   A. 0

   B. 1

   C. $\frac{1}{e}$

   D. e

   E. The integral diverges.

13. A company produces x sprockets and y pistons. The sprockets may be sold for $75 - x$ dollars each and the pistons may be sold for $200 - 2y + x$ dollars each. The cost of manufacturing a sprocket is 10 dollars each and the cost for manufacturing a piston is 40 dollars each. To the nearest dollar, what is the largest possible profit the company can attain by the production and sale of sprockets and pistons?

   A. \$3750

   B. \$4256

   C. \$5200

   D. \$6350

   E. \$9464

14. Find the maximum value of the function $f(x, y) = xe^{y^2/2}$ on the circle of radius 2 that is centered at the origin.

A. $\sqrt{2}$

B. 2

C. $e^{1/2}$

D. $e^{3/2}$

E. e^2

15. The velocity of a cyclist during an hour-long race is given by the function:

$$v(t) = 60te^{t/10} \text{ mi/hr}, \qquad 0 \leq t \leq 1$$

Assuming the cyclist starts from rest, what is the distance he traveled during the first 30 minutes of the race? Round your answer to two decimal places.

A. 3.12 miles

B. 6.80 miles

C. 7.75 miles

D. 16.04 miles

E. 32.08 miles