1. You must use a #2 pencil on the scantron answer sheet.
2. Fill in your name, your four digit section number, and your student identification number. If you do not know your section number, ask your instructor. (Leave the test/quiz number blank.) Sign your name.
3. There are 15 questions. Blacken in your choice of the correct answer in the spaces provided on the scantron answer sheet. **Only the scantron answer sheet will be graded**.
4. The exam is self-explanatory. Do not ask your instructor any questions about the exam problems.
5. Only one-line calculators (any brand) are allowed. Cell phones and PDA's may not be used as a calculator and must be put away during the exam. NO BOOKS OR PAPERS ARE ALLOWED.

CONSUMERS' AND PRODUCERS' SURPLUS

$$\text{CS} = \int_0^{q_0} D(q)dq - p_0q_0 \qquad \text{PS} = p_0q_0 - \int_0^{q_0} S(q)dq$$

TRAPEZOIDAL RULE

$$\int_a^b f(x)dx \equiv \frac{\Delta x}{2}\Big[f(x_1) + 2f(x_2) + 2f(x_3) + \cdots + 2f(x_n) + f(x_{n+1})\Big],$$

where $a = x_1, x_2, x_3, \ldots, x_{n+1} = b$ subdivides $[a, b]$ into n equal subintervals of length $\Delta x = \dfrac{b-a}{n}$.

THE SECOND DERIVATIVE TEST

Suppose f is a function of two variables x and y, and that all the second-order partial derivatives are continuous. Let

$$D = f_{xx}f_{yy} - (f_{xy})^2$$

and suppose (a, b) is a critical point of f.

1. If $D(a, b) < 0$, then f has a saddle point at (a, b).
2. If $D(a, b) > 0$ and $f_{xx}(a, b) < 0$, then f has a relative maximum at (a, b).
3. If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then f has a relative minimum at (a, b).
4. If $D(a, b) = 0$, the test is inconclusive.

VOLUME & SURFACE AREA

Right Circular Cylinder
$V = \pi r^2 h$
$$SA = \begin{cases} 2\pi r^2 + 2\pi r h \\ \pi r^2 + 2\pi r h \end{cases}$$

Sphere
$V = \frac{4}{3}\pi r^3$
$SA = 4\pi r^2$

Right Circular Cone
$V = \frac{1}{3}\pi r^2 h$
$SA = \pi r\sqrt{r^2 + h^2} + \pi r^2$

1. Evaluate the integral: $\int_2^{\infty} \frac{x^{-1/2}+x^{3/2}}{\sqrt{x}} dx$.

   A. 0

   B. 1

   C. ∞

   D. $ln2+2$

   E. e

2. Evaluate the partial derivative $f_x(x,y)$ at the point $(1,1)$.

$$f(x,y) = 3xy^2 + 3xe^{-x} + y^5 ln(x)$$

   A. $4+\frac{6}{e}$

   B. 4

   C. $11+\frac{3}{e}$

   D. 11

   E. $4+\frac{3}{e}$

3. Suppose $z = xy$, where $x = e^{-t/2}$ and $y = \ln(\sqrt{t})$. Use the chain rule to find $\frac{dz}{dt}$. Express your answer in terms of x, y and t.

   A. $-\frac{1}{2}ye^{-t/2} + \frac{x}{2t}$

   B. $ye^{-t/2} + \frac{x}{\sqrt{t}}$

   C. $-2ye^{-t/2} + \frac{x}{\sqrt{t}}$

   D. $-\frac{1}{2}ye^{-t/2} + \frac{x}{\sqrt{t}}$

   E. $-2ye^{-t/2} + \frac{x}{2t}$

4. Certain companies print and sell T-shirts with custom designs. An economist studying the supply of custom T-shirts gathers the data in the accompanying table, which lists the number of units q (in hundreds) of T-shirts that will be supplied to the market by producers at a price of p dollars per unit. Use this information together with the trapezoidal rule to estimate the producer's surplus when $700(q_0 = 7)$ T-shirts are supplied.

(100 units)	0	1	2	3	4	5	6	7
p (dollars/unit)	1.17	3.14	3.56	4.95	5.86	7.19	8.27	10.51

A. \$3,476

B. \$3,881

C. \$7,357

D. \$2,892

E. \$5,125

5. After t hours on the job, a factory worker's rate of production is $50te^{-0.2t}$ units per hour. How many units does the worker produce during his 8-hour work day? Round your answer to the nearest integer.

A. 81

B. 327

C. 594

D. 1200

E. 1512

6. A publisher speculates that $Q(x,y) = (1+x^{-1/3})y^{2/3}$ thousand textbooks will be sold per year if the price per book is x dollars and they get y schools to require custom editions. If the current price is \$150 and 30 schools require custom editions, use calculus to estimate the change in the number of books sold if the price is decreased \$5 and the publisher gets 2 more schools to require custom editions.

   A. 101

   B. 173

   C. 490

   D. 530

   E. 836

7. A T-shirt shop carries two competing shirts, one endorsed by Purdue University and the other by Indiana University. The owner of the store can obtain both types at a cost of \$2 per shirt and estimates that if Purdue shirts are sold for x dollars apiece and Indiana shirts for y dollars apiece, consumers will buy $50 - 20x$ Purdue shirts and $120 - 40y$ Indiana shirts each day. How should the owner price the shirts to generate the largest possible profit?

   A. $x = 2.5, y = 3$

   B. $x = 2.25, y = 2.5$

   C. $x = 2.2, y = 2.75$

   D. $x = 2.15, y = 2.5$

   E. $x = 2.4, y = 2.25$

8. Evaluate the definite integral:

$$\int_1^e \ln\sqrt{x}dx$$

A. $\frac{1}{\sqrt{e}} - 1$

B. $\frac{1}{2e} - \frac{1}{2}$

C. $\frac{1}{2}$

D. 2

E. $\frac{1}{4}$

9. The monthly demand of a product X is $f(x, y) = \frac{x+y}{x-y}$ units. Given $x(t) = 10 - t^2$ and $y(t) = 4t - 3$ are, respectively, the price of the product X and the price of a competing product Y, t months from now. At what rate will the monthly demand of product X be changing with respect to time two months from now?

A. 2

B. 8

C. 22

D. 44

E. 88

10. Find and classify the critical points of the function

$$f(x, y) = x^3 + 12xy + 12y^2 - 9x - 3.$$

A. Local maximum: none; Local minimum: $(3, -\frac{3}{2})$; Saddle Point: $(-1, \frac{1}{2})$.

B. Local maximum: $(-1, \frac{1}{2})$; Local minimum: $(3, -\frac{3}{2})$; Saddle Point: none.

C. Local maximum: $(-1, -\frac{3}{2})$; Local minimum: none; Saddle Point: $(3, \frac{1}{2})$.

D. Local maximum: none; Local minimum: none; Saddle Points: $(3, -\frac{3}{2})$, $(-1, \frac{1}{2})$.

E. Local maximum: $(3, -\frac{3}{2})$; Local minimum: none; Saddle Point: $(-1, \frac{1}{2})$.

11. A certain chemical reaction produces a compound X at the rate of:

$$t\sqrt{(t+1)^3} \text{ kg/hour},$$

where t is the time (in hours) from the start of the reaction. How much of the compound is produced during the first three hours of the reaction? Round your answer to the nearest kilogram.

A. 7 kg

B. 24 kg

C. 28 kg

D. 32 kg

E. 141 kg

12. Determine the y-intercept(s) of the level curve, where $f(x, y) = e^{\sqrt{x^2+y^2}}$ and $C = 2$.

   A. $(0, -ln2)$

   B. $(0, ln2)$

   C. $(0, -ln2), (0, ln2)$

   D. $(0, 2)$

   E. $(0, -2)$

13. Find $f_{xy}(x, y)$ if

$$f(x, y) = (x^3 + y^2)e^{-xy}$$

   A. $(x^4y + xy^3 - y^2 - 4x^3)e^{-xy}$

   B. $(x^4y + xy^3 - 3y^2 - 4x^3)e^{-xy}$

   C. $(3y^2 + 4x^3 - x^4y - xy^3)e^{-xy}$

   D. $(x^4y + xy^3 + y^2 - 4x^3)e^{-xy}$

   E. $(x^4y + xy^3 + 3y^2 - 4x^3)e^{-xy}$

14. We are tasked with constructing a rectangular box with a volume of 14 cubic feet. The material for the top costs 7 dollars per square foot, the material for the 4 sides costs 3 dollars per square foot, and the material for the bottom costs 5 dollars per square foot. To the nearest cent, what is the minimum cost for such a box?

A. \$82.99

B. \$600.00

C. \$122.83

D. \$82.21

E. \$131.74

15. Given the following three improper integrals:

$$\int_3^{+\infty} \frac{\ln x}{x} dx \qquad \int_3^{+\infty} \frac{\ln x}{x^2} dx \qquad \int_3^{+\infty} \frac{1}{x \ln x} dx$$

Which of the following statements are true?

I. One of the above integrals diverges. Two of above integrals converge.

II. The value of exactly one integral from the above integrals is a positive real number (Note: $+\infty$ is not a real number).

III. The value of exactly one integral from the above integrals is 0.

A. I, II, and III are true.

B. Only I and II are true.

C. Only I is true.

D. Only II is true.

E. Only III is true.