1. You must use a #2 pencil on the scantron answer sheet.
2. Fill in your name, your four digit section number, "01" for the Test/Quiz Number, and your student identification number. Make sure to blacken in the appropriate spaces. If you do not know your section number, ask your instructor. Sign your name.
3. There are 15 questions. Blacken in your choice of the correct answer in the spaces provided on the scantron answer sheet. **Only the scantron answer sheet will be graded**.
4. The exam is self-explanatory. Do not ask your instructor any questions about the exam problems.
5. Only one-line calculators are allowed. Cell phones and PDA's may not be used during the exam and must be put away. NO BOOKS OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

LAGRANGE EQUATIONS

For the function $f(x, y)$ subject to the constraint $g(x, y) = k$, the Lagrange equations are

$$f_x = \lambda g_x \qquad f_y = \lambda g_y \qquad g(x,y) = k$$

GEOMETRIC SERIES

If $0 < |r| < 1$, then

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

TAYLOR SERIES

The Taylor series of $f(x)$ about $x = a$ is the power series

$$\sum_{n=0}^{\infty} a_n(x-a)^n \quad \text{where} \quad a_n = \frac{f^{(n)}(a)}{n!}$$

Examples:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ for } -\infty < x < \infty; \qquad \ln x = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}(x-1)^n, \text{ for } 0 < x \le 2$$

VOLUME & SURFACE AREA

Right Circular Cylinder

$V = \pi r^2 h$

$$SA = \begin{cases} 2\pi r^2 + 2\pi r h \\ \pi r^2 + 2\pi r h \end{cases}$$

Sphere

$V = \frac{4}{3}\pi r^3$

$SA = 4\pi r^2$

Right Circular Cone

$V = \frac{1}{3}\pi r^2 h$

$SA = \pi r\sqrt{r^2 + h^2} + \pi r^2$

1. A company's total profit from selling x thousand units of Product A and y thousand units of Product B is:

$$P(x, y) = 6.3x + 8.4y$$

measured in millions. The quantities produced must satisfy the production possibilities curve:

$$x^2 + y^2 = 144$$

Assuming a maximum exists, how many units of each product should the company produce so that their profit is maximized?

A. $x = 4800$, $y = 7200$

B. $x = 9600$, $y = 7200$

C. $x = 6650$, $y = 10000$

D. $x = 7200$, $y = 9600$

E. $x = 10000$, $y = 6650$

2. Use summation notation to write the given series in compact form.

$$\frac{1}{6} - \frac{2}{7} + \frac{3}{8} - \frac{4}{9} + \dots$$

A. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{3n - 1}$

B. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n} n}{3n + 1}$

C. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n + 5}$

D. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^2}{n(n - 1)^2}$

E. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n} n}{2n + 3}$

3. Evaluate $\int_1^3 \int_0^2 x^3 y^2 dy dx$.

   A. 104/3

   B. 968/5

   C. 484/15

   D. 160/3

   E. 121/3

4. Evaluate the series $\sum_{n=3}^{\infty} \frac{2^{2n-4}}{5^{n-2}}$.

   A. 4

   B. 4/5

   C. 5

   D. 5/3

   E. The series diverges.

5. Determine the interval of convergence for the power series

$$\sum_{n=0}^{\infty} (-2)^{-n} x^n$$

   A. $x = 0$

   B. $-\frac{1}{2} < x < \frac{1}{2}$

   C. $-1 < x < 1$

   D. $-2 < x < 2$

   E. $-\infty < x < \infty$

6. Compute $\int_0^{\ln 2} \int_y^{3y} e^{x-2y} dxdy$.

A. $-\ln 2$

B. $1/\ln 2$

C. 2

D. $-1/2$

E. $1/2$

7. Let S_k represent the sum of the first k terms of the series $\sum_{n=1}^{\infty} \frac{(-1)^n(n)}{2^{n-3}}$. Find $S_7 - S_4$.

A. $-\frac{15}{16}$

B. $\frac{17}{16}$

C. 2

D. -4

E. 0

8. Which of the following series converge?

I. $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{(-4)^{2n}}$

II. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{2n-1}}{(n+1)7^{n+3}}$

III. $\displaystyle\sum_{n=2}^{\infty} \frac{5n^3}{(n-1)!}$

A. III

B. I, II

C. I, III

D. II, III

E. I, II, III

9. Use the method of Lagrange multipliers to find the maximum and minimum values of $f(x, y) = x^2 + y^2 + 4x + 2y + 1$ subject to the constraint $x^2 + y^2 = 5$.

A. max. = 25, min. = 0

B. max. = 6, min. = 6

C. max. = 16, min. = -4

D. max. = 15, min. = -2

E. max. = $26 + 10\sqrt{5}$, min. = $26 - 10\sqrt{5}$

10. Evaluate $\iint_R (3y - 2x)dA$ for the region R bounded by

$$y = \sqrt{x} \qquad \text{and} \qquad y = x^2$$

A. 4/5

B. 9/20

C. 2

D. 1/5

E. 3/20

11. Determine the radius of convergence for $\sum_{n=0}^{\infty} \frac{(-2x)^{2n}}{5}$.

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. $\frac{5}{4}$

D. $\frac{5}{2}$

E. ∞

12. You are building a barn, with no floor, in the shape of a rectangular box with a square base. The roof material costs \$15 per m^2, the sides and back material costs \$12 per m^2, and the front material costs \$20 per m^2. The volume of the barn will be 16,000 m^3. What dimensions minimize the total cost?

A. 17.28 m by 17.28 m by 53.57 m

B. 31.49 m by 31.49 m by 5.40 m

C. 20.47 m by 20.47 m by 38.20 m

D. 28.94 m by 28.94 m by 19.10 m

E. 31.03 m by 31.03 m by 16.62 m

13. Find a power series for the function $f(x) = \dfrac{10x^2}{5+x}$

A. $\displaystyle\sum_{n=2}^{\infty} \frac{2}{5^{n-2}} x^n$

B. $\displaystyle\sum_{n=2}^{\infty} 2(-1)^n x^n$

C. $\displaystyle\sum_{n=2}^{\infty} \frac{2(-1)^n}{5^{n-2}} x^n$

D. $\displaystyle\sum_{n=2}^{\infty} 10(-1)^n x^n$

E. $\displaystyle\sum_{n=2}^{\infty} \frac{2}{5^n} x^n$

14. A certain reactor runs on Obtainium, which is highly radioactive and degrades over time. Each year on the 1st of January, workers add 20 kilograms of Obtainium to the reactor. The fraction of Obtainium added to the reactor that remains after t years is $e^{-0.4t}$. In the long run, how many kilograms of Obtainium will be in the reactor on the 31st of December, immediately prior to adding more Obtainium to the reactor? (Give your answer to the nearest kilogram)

A. 23

B. 41

C. 61

D. 81

E. 101

15. Find the Taylor series for $f(x) = \dfrac{1}{3-x}$ about $x = -1$.

A. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^n}(x-1)^n$

B. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^{n+1}}(x-1)^n$

C. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^{n+2}}(x+1)^n$

D. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{4^n}(x+1)^n$

E. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{4^{n+1}}(x+1)^n$