MA 261

Exam 2

November 7, 2005

Instructions:
1. This exam contains 10 problems worth 10 points each.
2. Please supply all information requested above and on the mark-sense sheet.
3. Work only in the space provided, or on the backside of the pages. Mark your answers clearly on the scantron. Also circle your choice for each problem in this booklet.
4. No books, notes, or calculator, please.

Key: DDEC AEAD DC
1. By using a linear approximation of \( f(x, y) = \sqrt{x^2 + y} \) at \((4, 9)\), compute the approximate value of \( f(5, 8) \).

A. 5.2  
B. 5.3  
C. 5.5  
D. 5.7  
E. 5.9

2. If \( xz^3 - xyz = 4 \), find \( \frac{\partial z}{\partial x} \).

A. \( \frac{\partial z}{\partial x} = \frac{xz}{z^3 - y^2} \)  
B. \( \frac{\partial z}{\partial x} = \frac{3xz^2 - xy}{z^3 - yz} \)  
C. \( \frac{\partial z}{\partial x} = 2x + xy \)  
D. \( \frac{\partial z}{\partial x} = \frac{yz - z^3}{3xz^2 - xy} \)  
E. \( \frac{\partial z}{\partial x} = z^3 - yz \)
3. The directional derivative of \( f(x, y) = x^3 e^{-2y} \) in the direction of greatest increase of \( f \) at the point \((1, 0)\) is

A. 6  
B. 5  
C. \(\sqrt{5}\)  
D. 13  
E. \(\sqrt{13}\)

4. For the function \( f(x, y) = x^3 + 2y^2 + xy - 2x + 5y \), the point \((-1, -1)\) yields

A. a local minimum  
B. a local maximum  
C. a saddle point  
D. \( \nabla f(-1, -1) \neq 0 \)  
E. The Second Derivative Test gives no information at \((-1, -1)\)
5. Use the method of reversing the order of integration to compute
\[ \int_0^1 \int_{2x}^2 e^{y^2} dy dx. \]

A. \(\frac{1}{4}(e^4 - 1)\)
B. \(\frac{1}{2}(e^2 - 1)\)
C. \(\frac{1}{6}(e^3 - 1)\)
D. \(\frac{1}{2}(e^2 - e)\)
E. \(\frac{1}{4}(e^2 - e)\)

6. A flat plate of constant density occupies the region in the xy-plane bounded by the curves \(x = 0\) and \(x = \sqrt{1 - y^2}\). If \(\overline{x}, \overline{y}\) is the center of mass, then \(\overline{x}\) equals

A. \(\frac{2}{3\pi}\)
B. \(\frac{1}{2}\)
C. \(\frac{2}{\pi}\)
D. \(\frac{3}{2\pi}\)
E. \(\frac{4}{3\pi}\)
7. Find the maximum of \( f(x, y) = x + y \) on the curve defined by \( x^2 + 2y^2 = 6 \).

A. 3
B. 6
C. 5
D. \( \frac{3}{2} \)
E. \( \frac{5}{2} \)

8. Find the area of the part of the surface \( z = 2\sqrt{x^2 + y^2} \) that lies above the disk \((x - 1)^2 + y^2 < 1\).

A. \( \sqrt{7} \pi \)
B. \( 2\pi \)
C. \( 4\pi \)
D. \( \sqrt{5} \pi \)
E. \( \frac{\sqrt{8} \pi}{3} \)
9. Which of the following integrals equals the volume of the solid bounded by \( x = 0, \ y = 0, \ z = 0 \) and \( 2x + y + z = 4 \).

A. \[ \int_0^4 \int_0^4 \int_0^2 1 \, dx \, dy \, dz \]
B. \[ \int_0^2 \int_0^{4-2x} \int_0^{4-y} 1 \, dz \, dy \, dx \]
C. \[ \int_0^4 \int_0^{2x} \int_0^{4-y} 1 \, dz \, dy \, dx \]
D. \[ \int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} 1 \, dz \, dy \, dx \]
E. \[ \int_0^2 \int_0^1 \int_0^1 1 \, dz \, dx \, dy \]

10. Which of the following integrals in spherical coordinates equals \( \iiint_E z \, d\tau \), where \( E \) is the solid in the first octant satisfying \( x^2 + y^2 + z^2 < 9 \) and \( z < \sqrt{x^2 + y^2} \).

A. \[ \int_0^{\pi/2} \int_0^{\pi/2} \int_0^3 \rho^2 \cos \phi \sin \phi \, d\rho \, d\phi \, d\theta \]
B. \[ \int_0^{\pi/2} \int_0^{\pi} \int_0^3 \rho^3 \sin^2 \phi \, d\rho \, d\phi \, d\theta \]
C. \[ \int_0^{\pi/2} \int_0^{\pi/2} \int_0^3 \rho^3 \cos \phi \sin \phi \, d\rho \, d\phi \, d\theta \]
D. \[ \int_0^{2\pi} \int_0^{\pi/2} \int_0^3 \rho \cos \phi \, d\rho \, d\phi \, d\theta \]
E. \[ \int_0^{\pi/2} \int_0^{\pi/2} \int_0^3 \rho^3 \cos \phi \, d\rho \, d\phi \, d\theta \]