INSTRUCTIONS

1. There are 7 different test pages (including this cover page). Make sure you have a complete test.

2. Fill in the above items in print. I.D.# is your 9 digit ID (probably your social security number). Also write your name at the top of pages 2–7.

3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet for the first 7 questions. Partial credit will be given for work on the last 3 questions.

4. No books, notes or calculators may be used on this exam.

5. Each problem is worth 10 points. The maximum possible score is 100 points.

6. Using a #2 pencil, fill in each of the following items on your answer sheet:

   (a) On the top left side, write your name (last name, first name), and fill in the little circles.

   (b) On the bottom left side, under SECTION, write in your division and section number and fill in the little circles. (For example, for division 9 section 1, write 0901. For example, for division 38 section 2, write 3802).

   (c) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your student ID number, and fill in the little circles.

   (d) Using a #2 pencil, put your answers to questions 1–7 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.

   (e) Sign your answer sheet.

7. After you have finished the exam, hand in your answer sheet and your test booklet to your instructor.
1. Let \( \vec{u} = a\vec{i} + b\vec{j} \) be a unit vector. If \( f(x, y) = 2x + x^2y + \sin y \), what is the directional derivative \( D_{\vec{u}}f \) at a point \((x, y)\) in terms of \(a\) and \(b\).

A. \((2 + 2x)a + (\cos y)b\)

B. \((2 + 2xy)a + (\cos y)b\)

C. \((2 + 2xy)b + (x^2 + \cos y)a\)

D. \((2 + 2xy)b + (x^2 + \sin y)b\)

E. \((2 + 2xy)a + (x^2 + \cos y)b\)

2. An equation of the tangent plane to the graph of \( f(x, y) = x^2 + 4y^2 \) at the point \((2, 1, 8)\) is:

A. \(z = -8 + 2x + 4y\)

B. \(z = -8 + 4x + 8y\)

C. \(z = 8 + 2x + 4y\)

D. \(z = 8 + 4x + 8y\)

E. \(z = 8 + (x - 2) + (y - 1)\)
3. If \( f(3, 4) = 10 \) and \( df = 3dx + dy \) at (3, 4), what is the approximate value of \( f(3\frac{1}{2}, 3\frac{1}{2}) \) using the method of differentials?

A. \( 10\frac{5}{6} \)
B. 11
C. \( 11\frac{1}{6} \)
D. \( 11\frac{1}{2} \)
E. 12

4. What is the maximum value of the function \( x + 2y \) on the ellipse given by \( 4x^2 + 2y^2 = 9 \)?

A. 4
B. \( \frac{9}{2} \)
C. \( \sqrt{20} \)
D. \( 1 + 2\sqrt{3} \)
E. \( \frac{5}{2} \)
5. Find the volume of the solid region bounded by the planes \( x = 1, \ y = x, \ y = 2x, \ z = 0, \) and \( z = x. \)

A. 1  
B. \( \frac{1}{3} \)  
C. \( \frac{1}{2} \)  
D. \( \frac{2}{3} \)  
E. \( \frac{1}{6} \)

6. Evaluate \( \iint_D y \, dA \) where \( D \) is the region in the \( xy \)-plane bounded by \( y = 0 \) and \( y = \sqrt{1 - x^2}. \)

A. \( \frac{1}{3} \)  
B. \( \frac{\pi}{3} \)  
C. \( \frac{\pi}{2} \)  
D. 1  
E. \( \frac{2}{3} \)
7. If \( R \) is the region in the first octant bounded by \( x^2 + y^2 + z = 1 \), then \( \iiint_R xz \, dz \, dy \, dx \) equals

A. \( \int_0^1 \int_0^{1-x^2} \int_0^{1-x^2-y^2} xz \, dz \, dy \, dx \)

B. \( \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^{1-x^2-y^2} xz \, dz \, dy \, dx \)

C. \( \int_0^1 \int_0^{1-x^2} \int_0^{1-x^2-y^2} xz \, dz \, dy \, dx \)

D. \( \int_0^1 \int_0^{1-x^2} \int_0^{1-x^2-y^2} xz \, dz \, dy \, dx \)

E. \( \int_0^1 \int_0^{1-x^2-y^2} xz \, dz \, dy \, dx \)

8. Let \( R \) denote the intersection of the unit ball \( x^2 + y^2 + z^2 < 1 \) with the solid cylinder of radius \( \frac{1}{2} \) about the \( z \)-axis. Express the volume of \( R \) as a triple integral in cylindrical coordinates. DO NOT EVALUATE THE INTEGRAL.
9. Find the surface area of the part of the paraboloid \( z = x^2 + y^2 \) that is below the plane \( z = 3 \).
10. The function $f(x, y) = \frac{x^3}{3} - 2xy + 2y^2 - 6x$ has two critical points. Find each one and determine if it is a relative maximum, relative minimum, or saddle point. Justify your answer.