MA 261 **FINAL EXAM** **SPRING 2003**

Name ______________________ Student ID Number ______________________

Lecturer ______________________ Recitation Instructor ______________________

Time of Recitation Class ______________________

Instructions:

1. The exam has 24 problems, problems 17–24 are worth 9 points each, the rest are worth 8 points each..
2. Please supply all information requested above.
3. Work only in the space provided, or on the backside of the pages.
4. No books, notes, or calculators are allowed.
5. Use a number 2 pencil on the answer sheet. Print your last name, first name, and fill in the little circles. Under "Section Number", print the division and section number of your recitation class and fill in the little circles. Similarly, fill in your student ID and fill in the little circles. Also, fill in your recitation instructor's name. Be sure to fill in the circles for each of the answers of the 24 exam questions.

Green's Theorem

$$\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

where C is positively oriented simple closed curve enclosing region D, and P, Q have continuous partial derivatives.

Divergence Theorem

$$\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_S \mathbf{F}\cdot\mathbf{n}\,dS = \iiint_E \text{div }\mathbf{F}\,dV$$

where E is simple solid region with boundary S given outward orientation, and component functions of $\mathbf{F}$ have continuous partial derivatives.

1. Find the area of a triangle with vertices $P = (1,1,1)$, $Q = (1,2,3)$, $R = (2,3,1)$.

A. $\sqrt{17}$

B. $\frac{\sqrt{17}}{2}$

C. $\sqrt{21}$

D. $\frac{\sqrt{21}}{2}$

E. $\sqrt{11}$

2. Find vector equation for the line containing $P = (-1,1,0)$ and $Q = (-2,5,7)$.

A. $\mathbf{r} = \langle -1-t, 1+4t, 7t \rangle$

B. $\mathbf{r} = \langle -1-2t, 1+5t, 7t \rangle$

C. $\mathbf{r} = \langle -2-t, 5+t, 7 \rangle$

D. $\mathbf{r} = \langle -1+2t, 1-5t, -7t \rangle$

E. $\mathbf{r} = \langle -2+t, 5-t, 7 \rangle$

3. Find the intersection of the planes $x+2y+3z=0$ and $2x+y+z=0$.

A. $y = x,\ z = -x$

B. $y = -x,\ z = -x$

C. $y = x,\ z = -3x$

D. $y = -5x,\ z = 3x$

E. $y = 5x,\ z = -3x$

4. Let $w = e^{uv}$, $u = x + y$, $v = v(x, y)$ with $v(x, 0) = 0$ for all x. If
$w_y = v(x, y)e^{(x+y)v(x,y)} + (x + y)xe^{(x+y)v(x,y)}$
then $v(x, y) =$

A. $\dfrac{y}{x}$
B. xy
C. $\dfrac{yx^2}{2}$
D. y^2x
E. ye^x

5. The function $f(x, y) = x^3 + y^3 - 3x^2y$ has how many critical points?

A. None
B. Two
C. Three
D. Four
E. One

6. At which point(s) on the surface $z = \frac{1}{3}(x^3 + y^3)$ is the tangent plane parallel to $x - y - z = 4$?

A. At no point
B. $(1, 1)$ and $(-1, -1)$
C. $(1, -1)$ and $(-1, 1)$
D. $(1, 1)$
E. $(-1, -1)$

7. The maximum and minimum values of the function $f(x, y) = x^2 + y^2 - 2(x + y)$ on the disk $x^2 + y^2 \leq 8$ are

A. $14, 0$

B. $14, -2$

C. $16, -2$

D. $18, -2$

E. $18, -4$

8. Find the point on the surface $z = 3x^2 - y^2$, where the vector $-6\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ is normal to the surface.

A. $(-1, -1, 2)$

B. $(-1, 1, 2)$

C. $(1, -1, 2)$

D. $(1, 1, 2)$

E. $(1, 2, -1)$

9. What is the largest directional derivative of the function $f(x, y) = x^2 + xy + 2y^2$ at the point $(2, 1)$?

A. 1

B. $\sqrt{21}$

C. $\sqrt{41}$

D. $\sqrt{61}$

E. 9

10. Evaluate $\int_0^1 \int_x^1 e^{\frac{x}{y}} dydx$

A. $e-1$

B. $1-e$

C. $\frac{1}{2}e$

D. $e-\frac{1}{2}$

E. $\frac{1}{2}(e-1)$

11. The double integral $\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} x^2y^3dxdy$ when converted to polar coordinates becomes.

A. $\int_0^{\frac{\pi}{2}} \int_0^2 r^5 \cos^2\theta \sin^3\theta dr d\theta$

B. $\int_0^{\frac{\pi}{2}} \int_0^2 r^6 \cos^2\theta \sin^3\theta dr d\theta$

C. $\int_0^{\frac{\pi}{2}} \int_1^2 r^6 \cos^2\theta \sin^3\theta dr d\theta$

D. $\int_0^{\frac{\pi}{4}} \int_1^2 r^6 \cos^2\theta \sin^3\theta dr d\theta$

E. $\int_0^{\frac{\pi}{4}} \int_0^2 r^6 \cos^2\theta \sin^3\theta dr d\theta$

12. Given $\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x,y,z)dzdydx$, find the limits of integration when the order of integration is changed to $dydxdz$.

A. $\int_0^1 \int_0^{(1-z)^2} \int_{\sqrt{x}}^{1-z} f(x,y,z)dydxdz$

B. $\int_0^1 \int_0^{(1-z)^2} \int_0^{1-z} f(x,y,z)dydxdz$

C. $\int_0^1 \int_0^{(1-z)^2} \int_0^{\sqrt{x}} f(x,y,z)dydxdz$

D. $\int_0^1 \int_z^1 \int_{\sqrt{x}}^1 f(x,y,z)dydxdz$

E. $\int_0^1 \int_0^{1-z} \int_0^{\sqrt{x}} f(x,y,z)dydxdz$

13. Let $\mathbf{F} = \langle yz^2, x^2z, xy^2 \rangle$, curl($\mathbf{F}$) at $(x, y, z) = (1, 2, 3)$ is

A. $\langle 3, -8, 3 \rangle$

B. $\langle 3, 8, 3 \rangle$

C. $\langle 3, 8, -3 \rangle$

D. $\langle -3, 8, 3 \rangle$

E. $\langle 3, -8, -3 \rangle$

14. Which of the following make sense for a function f?

1. div(grad(f))
2. grad(div(f))
3. curl(grad(f))
4. grad(curl(f))
5. div(curl(f))
6. curl(div(f))

A. 1, 3, 5

B. 2, 4, 6

C. 1, 3

D. 4, 6

E. 2, 5

15. Evaluate $\int_C xy\,ds$ where $C = \{(x, y) | x^2 + y^2 = 4, x \geq 0, y \geq 0\}$ oriented counter-clockwise.

A. 0

B. 1

C. 2

D. 4

E. 8

16. Evaluate $\int_C y\,dx + x^2 y\,dy$ where C consists of the line segments from $(0,0)$ to $(1,0)$ and from $(1,0)$ to $(1,2)$.

A. 0

B. $\frac{2}{3}$

C. 2

D. $\frac{5}{6}$

E. 3

17. Evaluate $\int_C (y+1)dx + xdy$ where

$C : \mathbf{r}(t) = te^{\sqrt{t}-1}\mathbf{i} + t^2 \sin\left(\frac{\pi t}{2}\right)\mathbf{j},\ 0 \le t \le 1.$

A. 0

B. 1

C. e

D. 2

E. $-e+1$

18. If C goes from $(1,0)$ counterclockwise once around the circle $x^2+y^2=1$, then $\int_C (x^2+2xy)dx + (x^2+2x+y)dy =$

A. 4π

B. -4π

C. -2π

D. 0

E. 2π

19. Find the area of the parametric surface $\mathbf{r}(u,v) = u\cos v\mathbf{i} + u\sin v\mathbf{j} + v\mathbf{k}$, $0 \le u \le \pi$, $0 \le v \le u$.

A. $\frac{2}{3}(1+\pi^2)^{\frac{3}{2}}$

B. $\frac{1}{3}((1+\pi^2)^{\frac{3}{2}} - 1)$

C. $\frac{1}{2}(\sqrt{1+\pi^2} - 1)$

D. $\frac{1}{5}((1+\pi^2)^{\frac{3}{2}} - 1)$

E. $\frac{1}{5}(1+\pi^2)^{\frac{3}{2}}$

20. If $\mathbf{F}(x,y,z) = P(x,y,z)\mathbf{i} + Q(x,y,z)\mathbf{j} + R(x,y,z)\mathbf{k}$ is defined everywhere in $\mathbf{R}^3$ and curl $\mathbf{F} = 0$ then which of the following are true?

A. Div $\mathbf{F} = 0$

B. $\nabla(\text{Div } \mathbf{F}) = 0$

C. $\mathbf{F} = \nabla f$ for some function f

D. both A and C

E. both B and C

21. Find the surface area of the cone $z = \sqrt{x^2+y^2}$ with $0 \le z \le 2$.

A. $\sqrt{2}\pi$

B. $2\sqrt{2}\pi$

C. $3\sqrt{2}\pi$

D. $4\sqrt{2}\pi$

E. $5\sqrt{2}\pi$

22. Evaluate $\iint_S z\,dS$, where S is the part of the plane $2x+2y+z=4$ in the first octant.

A. $\frac{8}{3}$
B. 6
C. 8
D. $-\frac{8}{3}$
E. $\frac{16\sqrt{2}}{3}$

23. Let $\mathbf{F}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ and S be the surface $z=1-x^2-y^2$ with $z\geq 0$. Evaluate $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$, where $\mathbf{n}$ points upward.

A. π
B. 2π
C. $\frac{4\pi}{3}$
D. $\frac{5\pi}{3}$
E. $\frac{3}{2}\pi$

24. Evaluate $\iint_S \mathbf{F}\cdot d\mathbf{S}$, where $\mathbf{F}=xz\mathbf{i}-2y\mathbf{j}+3z\mathbf{k}$ and S is the sphere $x^2+y^2+z^2=4$ with normal pointing outward.

A. $\frac{32}{3}\pi$
B. $\frac{44}{3}\pi$
C. 12π
D. 11π
E. 10π