MA 261 | FINAL EXAM | May 1, 2006

Name: ______________________________

Student I.D. #: ______________________________

Lecturer: ______________________________

Recitation Instructor: ______________________________

Div: ________________ Sec: ________________

Instructions:

1. This exam contains 25 problems worth 8 points each.
2. Please supply all information requested above and on the scantron.
3. Work only in the space provided, or on the backside of the pages. Mark your answers clearly on the scantron. Also circle your choice for each problem in this booklet. Make sure your test cover page and scantron are the same color.
4. No books, notes, or calculator, please.

1. Find the volume of the parallelepiped with edges determined by the vectors $\vec{a} = 3\vec{i} + 2\vec{j} + \vec{k}$, $\vec{b} = \vec{j} + 2\vec{k}$, and $\vec{c} = \vec{i} - 2\vec{j} + \vec{k}$.

A. 3

B. 6

C. 9

D. 18

E. 24

2. Find parametric equations for the line passing through $P = (2, 1, -1)$, and perpendicular to the plane

$$4x + 2y + 3z = 8.$$

A. $x = t + 4, y = t, z = -t$

B. $x = 4t + 2, y = 2t + 1, z = 3t - 1$

C. $\dfrac{x-2}{4} = \dfrac{y-1}{2} = \dfrac{z+1}{3}$

D. $\dfrac{x-4}{2} = \dfrac{y-3}{9} = \dfrac{z-3}{-1}$

E. $x = 4t - 2,\ y = 2t - 1,\ z = -3t + 1$

3. Find the equation in spherical coordinates for $x^2 + y^2 = x$.

A. $\rho = \sin\phi\cos\theta$

B. $\rho\sin\phi = \sin^2\phi\cos\theta$

C. $\rho = \sin\phi\cos\phi$

D. $\rho^2 = \rho\cos\phi$

E. $\rho^2\sin^2\phi = \rho\sin\phi\cos\theta$

4. Find symmetric equations for the line through the origin and perpendicular to the lines

$$\frac{x}{2} = y = -z \quad \text{and} \quad x = \frac{y}{2} = \frac{z}{3}.$$

A. $\frac{x}{5} = \frac{y}{-7} = \frac{z}{3}$

B. $\frac{x}{2} = \frac{y}{-3} = z$

C. $\frac{x}{2} = \frac{y}{3} = z$

D. $\frac{x}{5} = \frac{y}{7} = z$

E. $\frac{x}{2} = \frac{y}{2} = \frac{z}{3}$

5. A vector parallel to the tangent to the curve $x = 3t^{\frac{4}{3}}$, $y = 2t^3 - 1$, $z = \frac{2}{t^2}$ at the point $P(3, -3, 2)$ on the curve is:

A. $-2\vec{i} + 3\vec{j} - 2\vec{k}$

B. $-2\vec{i} + 3\vec{j} + 2\vec{k}$

C. $-2\vec{i} + 3\vec{j} + \vec{k}$

D. $-2\vec{i} - 3\vec{j} - \vec{k}$

E. $-4\vec{i} - 3\vec{j} - 2\vec{k}$

6. The curve given parametrically by:

$$x = \cos^3 t,\ y = 3,\ z = \sin^3 t,\ 0 \le t \le \frac{\pi}{2}$$

has arclength equal to:

A. $\frac{3}{2}$

B. $\frac{3\pi}{2}$

C. 3

D. $\frac{9\pi}{2}$

E. 9

7. Evaluate $\iint_R \frac{2x}{y} dA$ where R is the region bounded by the lines $y = \dfrac{x}{2}$, $y = x$, and between $y = 2$ and $y = 4$.

A. $2 \ln 2$

B. $12 \ln 2$

C. $\dfrac{3}{4}$

D. 18

E. 36

8. If $f(x, y) = e^{xy} \ln y$, then $f_y(2, y) =$

A. $\dfrac{e^{2y}}{y}$

B. $e^{2y}(\dfrac{1}{y} + 2 \ln y)$

C. $e^{2y}(\dfrac{1}{y} + \ln y)$

D. $\dfrac{2e^{2y}}{y}$

E. $e^{2y}(\dfrac{1}{2} + 2 \ln y)$

9. One vector perpendicular to the plane that is tangent to the surface $x^2 + y^2 + z = 9$ at the point $(1, 2, 4)$ on the surface is:

A. $2\vec{i} + 2\vec{j} + \vec{k}$

B. $2\vec{i} + 4\vec{j} + \vec{k}$

C. $-2\vec{i} - 4\vec{j} + \vec{k}$

D. $\vec{i} + \vec{j} - 4\vec{k}$

E. $\vec{i} + \vec{j} + 4\vec{k}$

10. Suppose $z = f(x, y)$, where $x = e^t$ and $y = 2s + 3t + 2$. Given that $\dfrac{\partial z}{\partial x} = 2xy$ and $\dfrac{\partial z}{\partial y} = x^2$, find $\frac{\partial z}{\partial t}$ when $s = 0$ and $t = 0$.

A. 5

B. 6

C. 7

D. 11

E. $6e^2$

11. Find the direction in which the function $z = x^2 + 3xy - \frac{1}{2}\, y^2$ is increasing most rapidly at $(-1, -1)$.

A. 29

B. $5\vec{\imath} + 2\vec{\jmath} - \vec{k}$

C. $-5\vec{\imath} - 2\vec{\jmath}$

D. $2\vec{\imath} - 5\vec{\jmath}$

E. $\sqrt{29}$

12. The function $f(x, y) = 6x^2 - 2x^3 + y^3 + 3y^2$ has how many critical points?

A. None

B. One

C. Two

D. Three

E. More than three

13. If $\vec{r}(t) = t^2\vec{i} + 3t\vec{j} + \vec{k}$ is the position of a moving particle at time t, then the speed of the particle at $t = 1$ is:

A. $\vec{i} + 3\vec{j} + \vec{k}$

B. $2\vec{i} + 3\vec{j}$

C. $\sqrt{11}$

D. $\sqrt{13}$

E. $\sqrt{14}$

14. Find the maximum value of $x^2 + y^2$ subject to the constraint $x^2 - 2x + y^2 - 4y = 0$.

A. 2

B. 4

C. 10

D. 16

E. 20

15. Let R be the region in the xy–plane bounded by $y = x$, $y = -x$ and $y = \sqrt{4 - x^2}$. Evaluate the integral

$$\iint_R y dA.$$

A. $\frac{8\sqrt{3}}{2}$

B. $\frac{8}{3\sqrt{2}}$

C. $\frac{4}{\sqrt{2}}$

D. $\frac{8\sqrt{2}}{3}$

E. $4\sqrt{2}$

16. A lamina in the xy–plane bounded by $y = 0$, $x = 0$ and $2x + y = 2$ has mass density at (x, y) equal to the distance to the x–axis. Find the mass of the lamina.

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{4}{3}$

D. 2

E. 1

17. Find the surface area of the part of the surface $z = x^2 + y^2$ below the plane $z = 9$.

A. $\frac{\pi}{4}(3\sqrt{3} - 1)$

B. $\frac{\pi}{4}(3\sqrt{3} - 2\sqrt{2})$

C. $\frac{\pi}{6}(37^{3/2} - 1)$

D. $\frac{\pi}{6}(29^{3/2} - 1)$

E. $\frac{\pi}{6}(23^{3/2} - 1)$

18. Find a, b such that

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^2 z^2 x\,dz\,dy\,dx = \int_0^2 \int_0^a \int_0^b z^2 x\,dx\,dy\,dz.$$

A. $a = 3 \quad b = x$

B. $a = \sqrt{9 - z^2} \quad b = 3$

C. $a = 3 \quad b = \sqrt{9 - y^2}$

D. $a = z \quad b = 3$

E. $a = 3 \quad b = \sqrt{9 - x^2}$

19. $\lim\limits_{(x,y)\to(0,0)} e^{\left(\frac{-1}{x^2+y^2}\right)} \cos(x^2+y^2)$

A. equals 0

B. equals -1

C. equals 1

D. equals e^{-1}

E. does not exist

20. If $\vec{F}(x,y,z) = (x\sin x + y)\vec{i} + xy\vec{j} + (yz+x)\vec{k}$, then curl $\vec{F}$ evaluated at $(\pi, 0, 2)$ equals

A. $\pi\vec{i} - \vec{j} + \vec{k}$

B. $2\vec{i} - \vec{j} - \vec{k}$

C. $2\vec{i} - \pi\vec{j} + \vec{k}$

D. $2\vec{i} - \vec{j} + \pi\vec{k}$

E. $2\vec{i} + \vec{j} + \vec{k}$

21. Evaluate $\int_C (2x+y)dx + xdy$

where $C: \vec{r}(t) = t^2(1+t)\vec{i} + \cos\left(\frac{\pi}{2}t^2\right)\vec{j},\ 0 \le t \le 1.$

A. 1

B. 2

C. 4

D. 5

E. 6

22. Consider the surface

$$S: x = u + v,\ \ y = u - v,\ \ z = u^2 + v^2.$$

Find the equation of the tangent plane to S at the point where $u = 1$ and $v = 0$.

A. $2x + 2y + z = 5$

B. $x + 2y - z = 0$

C. $2x + y - z = 0$

D. $x + y + z = 3$

E. $x + y - z = 1$

23. Evaluate $\iint_S (x^2 + y^2 + z^2) dS$ where S is the upper hemisphere of $x^2 + y^2 + z^2 = 2$.

A. 2π

B. 4π

C. 5π

D. 6π

E. 8π

24. Evaluate $\int_C 4y dx + 2x dy$ where C is the semi-circle $x^2 + y^2 = 1,\ y \geq 0$ oriented counterclockwise.

A. 0

B. π

C. 2π

D. $-\pi$

E. -2π

25. Calculate the surface integral $\iint_S \vec{F} \cdot \vec{n}\ dS$ where S is the sphere $x^2 + y^2 + z^2 = 2$ oriented by the outward normal and $\vec{F}(x, y, z) = 5x^3\vec{i} + 5y^3\vec{j} + 5z^3\vec{k}$.

A. $48\sqrt{2}\pi$

B. 16π

C. 24π

D. $25\sqrt{2}\pi$

E. 20π