INSTRUCTIONS:

1. **Do not open the exam booklet until you are instructed to do so.**
2. **Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
3. **MARK YOUR TEST NUMBER ON YOUR SCANTRON**
4. Once you are allowed to open the exam, make sure you have a complete test. There are 7 different test pages including this cover page.
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. No extra paper is allowed. Circle your answers on this test booklet.
6. There are eleven problems, each problem is worth 9 points and everyone gets one point. The maximum possible score is 100 points. No partial credit.
7. After you finish the exam, hand in your scantron and your test booklet to one of the proctors.

RULES REGARDING ACADEMIC DISHONESTY:

1. Do not leave the exam room during the first 20 minutes of the exam.
2. If you do not finish your exam in the first 50 minutes, you must wait until the end of the exam period to leave the room.
3. Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only your instructor.
4. Do not look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
5. Do not consult notes, books, calculators.
6. Do not handle phones or cameras, or any electronic device until after you have finished your exam, handed it in to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the proctors will collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

STUDENT ID NUMBER: _____SECTION NUMBER _____

RECITATION INSTRUCTOR: _____

1

1. If $y(x)$ is a solution to the initial value problem

$$y' = \sqrt{y}\,(4x+2)$$
$$y(0) = 1,$$

then $y(2)$ (the value of $y(x)$ when $x = 2$) is equal to

A. 49

B. $\dfrac{82}{9}$

C. $\dfrac{13}{4}$

D. 36

E. 10

2. Which of the following statements are correct?

I. If we solve the initial value problem $y' = y^2$, $y(0) = \dfrac{1}{2}$ we conclude that its solution $y(x)$ is continuous on the interval $(-1, 1)$, but not on the interval $(-3, 3)$.

II. The functions $y(x) = mx$ solve the initial value problem $xy' = y$, $y(0) = 0$, for any m. So this initial value does not have a unique solution.

III. The initial value problem $y' = xy$, $y(0) = 1$ has a unique solution $y(x)$ which is continuous for all x.

A. Only II and III are correct

B. Only II is correct

C. Only III is correct

D. Only I and II are correct

E. I, II and III are correct

3. Let $y(x)$ be the solution of the following initial value problem

$$xy' = y + \frac{3}{2}(x^2 y)^{\frac{1}{3}},$$
$$y(1) = 1.$$

Find the value $y(e^3)$.

A. e^3

B. $4e^3$

C. $8e^3$

D. $\frac{3}{2}e^3$

E. $\frac{1}{2}e^3$.

4. Let $y(x)$ be the solution of this initial value problem:

$$x^2 \frac{dy}{dx} = 2xy + 3,$$
$$y(1) = 0.$$

Find $y(2)$.

A. $\frac{7}{3}$

B. $\frac{7}{2}$

C. $-\frac{1}{2}$

D. $2e^2$

E. $\frac{1}{8}$.

5. Let $y(t)$ be the solution of the following initial value problem

$$y'' + 9y = 0,$$
$$y(0) = 0, \quad y'(0) = 2.$$

Find $y(\frac{\pi}{6})$.

A. $\dfrac{\sqrt{3}}{2}$

B. 0

C. $\dfrac{1}{\sqrt{2}}$

D. $\dfrac{2}{3}$

E. $2e^{\pi/6}$

6. Let $y(x)$ be the solution of the initial value problem

$$y' + y = y^2 e^x,$$
$$y(0) = 1.$$

Find $y(\frac{1}{2})$.

A. $-2e^{-\frac{1}{2}}$

B. $2e^{-\frac{1}{2}}$

C. $e^{-\frac{1}{2}}$

D. $-3e^{-\frac{1}{2}}$

E. $3e^{-\frac{1}{2}}$

7. The graph of a solution of the differential equation

$$(x^2y + x)dy + (xy^2 + y)dx = 0$$

passes through the point $(x, y) = (1, -1)$. If it also pass through the point $(x, y) = (2, a)$, then find a.

A. $a = 2$

B. $a = -\dfrac{1}{2}$

C. $a = \sqrt{2}$

D. $a = -\sqrt{2}$

E. $a = -1$

8. Classify the equilibrium solutions for the following differential equation,

$$y' = y^2(y^2 - 1).$$

A. $y(t) = -1$ is stable ; $y(t) = 0$ is semi-stable and $y(t) = 1$ is unstable

B. $y(t) = -1$ is unstable ; $y(t) = 0$ is semi-stable and $y(t) = 1$ is stable

C. $y(t) = -1$ is semi-stable ; $y(t) = 0$ is semi-stable and $y(t) = 1$ is stable

D. $y(t) = -1$ is stable ; $y(t) = 0$ is unstable and $y(t) = 1$ is stable

E. $y(t) = -1$ is stable ; $y(t) = 0$ is stable and $y(t) = 1$ is stable

9. The number $P(t)$ of bacteria in a culture obeys the equation $P(t)' = k\,P(t)$. After 2 hours there were 200 bacteria present. After 4 hours, 800 bacteria. Determine the initial number of bacteria.

A. 100

B. 50

C. 60

D. 80

E. 70

10. The system

$$x_1 + 3x_2 + 3x_3 = 1$$
$$2x_1 + 5x_2 + 4x_3 = a$$
$$2x_1 + 7x_2 + 8x_3 = b$$

will have no solutions if

A. $2a + b - 5 \neq 0$

B. $a + b - 5 \neq 0$

C. $3a + b - 5 \neq 0$
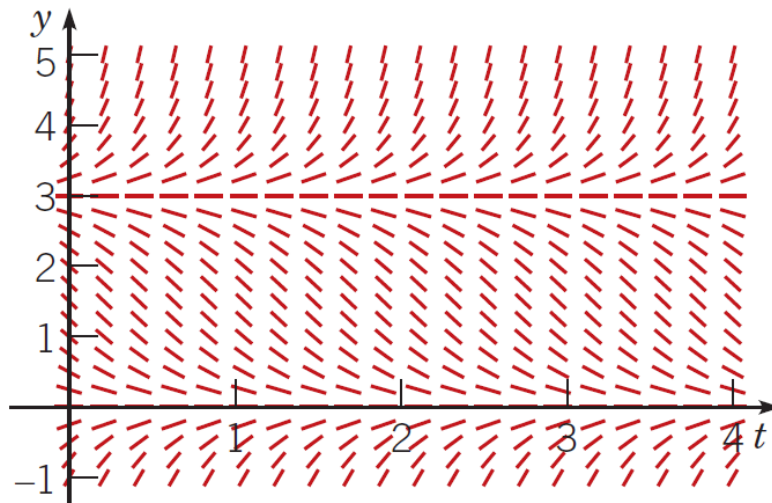
D. $a + b - 4 \neq 0$

E. $5a + b - 5 \neq 0$

FIGURE 1. Slope Field

11. Among the differential equations below, which one has a slope field which is similar to the one shown in Figure 1?

A. $y' = y(3 - y)$

B. $y' = y - 3$

C. $y' = y(3 + y)$

D. $y' = y(y - 3)$

E. $y' = y + 3$