MA 262, Spring 2018, Final exam
Version 01 (Green)

INSTRUCTIONS

1. **Switch off your phone upon entering the exam room.**
2. **Do not open the exam booklet until you are instructed to do so.**
3. **Before you open the booklet, fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
4. **MARK YOUR TEST NUMBER ON THE SCANTRON**
5. Once you are allowed to open the exam, make sure you have a complete test. There are 13 different test pages with a total of 25 problems, plus this cover page.
6. Do any necessary work for each problem on the space provided, or on the back of the pages of this booklet. Circle your answers in the booklet.
7. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

RULES REGARDING ACADEMIC DISHONESTY

1. Do not leave the exam during the first 20 minutes of the exam.
2. No talking. Do not seek or obtain any kind of help from anyone to answer the problems on the exam. If you need assistance, consult an instructor.
3. Do not look at the exam of another student. You may not compare answers with other students until your exam is finished and turned in, and then only after you have left the room.
4. Your bags must be closed throughout the exam period.
5. Notes, books, calculators and phones must be in your bags and cannot be used.
6. Do not handle phones or cameras or any other electronic device until you have finished and turned in your exam, and then only if you have left the room.
7. When time is called, all students must put down their writing instruments immediately.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME ____________________

STUDENT SIGNATURE ____________________

STUDENT PUID ____________________

SECTION NUMBER ____________________

RECITATION INSTRUCTOR ____________________

1. Solutions of $y^2 + \cos x + (2xy + \sin y)y' = 0$ satisfy:

A. $xy^2 + \sin x = C$

B. $xy^2 + \sin x - \cos y = C$

C. $\dfrac{y^3}{3} + \sin x + yx^2 - \cos y = C$

D. $\dfrac{y^3}{3} - \sin x \cos y + xy^2 = C$

E. $x^2y^2 + \sin x + \cos y = C$

2. The general solution of $y' + 3x^2y = e^{-x^3}$ is

A. $y = (x^2 + c)e^{x^3}$

B. $y = (x + c)e^{x^3}$

C. $y = (x^2 + c)e^{-x^3}$

D. $y = (x + c)e^{-x^3}$

E. $y = (x^2 + cx)e^{-x^3}$

3. The solution of $3y' = \dfrac{\sin x}{(1+y)^2}$ and $y(0) = 1$ is

A. $y = (9 - \cos x)^{\frac{1}{3}} - 1$

B. $y = (5 + 3\cos x)^{\frac{1}{3}} - 1$

C. $\mathrm{y} = 1 - (3 - 3\sin x)^{\frac{1}{3}}$

D. $\mathrm{y} = 1 - (1 - \cos x)^{\frac{1}{3}}$

E. $\mathrm{y} = 1 - (3 - 3\cos x)^{\frac{1}{3}}$

4. Using the substitution $u = \dfrac{1}{y}$, the differential equation $y' + \dfrac{3}{x}y = x^2y^2$ becomes

A. $u' + (3\ln x)u = x^2$

B. $u' - (3\ln x)u = -x^2$

C. $u' - \dfrac{3}{x}u = -x^2$

D. $u' + \dfrac{3}{x}u = -x^2$

E. $u' + \dfrac{3}{x^2}u = 1$

5. A tank initially contains 100 L of a solution in which is dissolved 50 g of chemical. Water flows into the tank at the rate of 4 L/min and the well -mixed solution flows out at the same rate. What is the amount of chemical (in grams) in the tank after 50 minutes?

A. $25e^{-4}$

B. $5e^{-4}$

C. $25e^{-2}$

D. $50e^{-2}$

E. $100 - 50e^{-2}$

6. Using the substitution $v = \dfrac{y}{x}$, the differential equation $y' = \dfrac{y - \sqrt{x^2 + y^2}}{x}$ becomes

A. $xv' = -\sqrt{1 + v^2}$

B. $xv' + xv = 1 - \sqrt{1 + v^2}$

C. $xv' + v = 1 - \sqrt{1 + v^2}$

D. $xv' = 1 - v\sqrt{1 + v^2}$

E. $xv' = v - \sqrt{1 + v^2}$

7. The general solution of $y'' + \frac{1}{x}y' = x^2$ is

A. $\mathrm{y} = \frac{x^4}{3} + c_1 x + c_2$

B. $\mathrm{y} = \frac{x^4}{3} + c_1 \ln x + c_2$

C. $\mathrm{y} = \frac{x^4}{16} + c_1 \ln x + c_2$

D. $\mathrm{y} = \frac{x^3}{4} + \frac{c_1}{x} + c_2 x$

E. $\mathrm{y} = \frac{x^4}{16} + c_1 \ln x + c_2 x$

8. For a real number a, consider the system of equations

$$\begin{array}{rcrcrcr} x & + & 2y & + & 3z & = & 2 \\ 4x & + & 5y & + & z & = & 3 \\ 4x & + & 5y & + & (a^2-3)z & = & a+1 \end{array}$$

Which of the following statements is true?

A. If $a = -2$ then the system is inconsistent.

B. If $a = 3$ then the system is inconsistent.

C. If $a = 1$ then the system has infinitely many solutions.

D. If $a = -1$ then the system has at least two distinct solutions.

E. If $a = 2$ then the system has a unique solution.

9. Let A and B be two 3×3 matrices with $\det(A) = 3$ and $\det(B) = -8$. Then $\det(2A\,B^{-1}) =$

A. 3

B. -6

C. $-\dfrac{3}{4}$

D. -3

E. $-\dfrac{3}{8}$

10. Let A be the matrix defined by

$$A = \begin{bmatrix} 0 & 0 & 2 \\ 1 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix}$$

The value of the $(2,1)$-element of A^{-1} is

A. $-\dfrac{1}{4}$

B. $-\dfrac{1}{2}$

C. 1

D. $\dfrac{3}{2}$

E. $\dfrac{5}{4}$

11. Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Which of the following statements is true?

A. The columns of A are linearly dependent

B. The matrix has determinant -1

C. The matrix is not invertible

D. $\text{colspace}(A) = \mathbb{R}^3$

E. The nullspace of A has dimension 1

12. Consider the following statements about vector spaces and matrices.

(i) If three vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ in $\mathbb{R}^4$ are linearly independent, then any two of them are also linearly independent.

(ii) If four vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ span $\mathbb{R}^3$, then any three of them also span $\mathbb{R}^3$.

(iii) The dimension of a subspace of $\mathbb{R}^n$ is at most n.

(iv) Every subspace of $\mathbb{R}^n$ contains at most n vectors.

The true statements are

A. (i) and (ii)

B. (iii) and (iv)

C. (i) and (iii)

D. (ii) and (iv)

E. (i), (iii) and (iv)

13. Consider the subspace U of $\mathbb{R}^5$ spanned by the set of vectors below.

$$\begin{bmatrix}2\\2\\2\\2\\2\end{bmatrix}, \quad \begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}, \quad \begin{bmatrix}0\\1\\0\\-1\\0\end{bmatrix}, \quad \begin{bmatrix}0\\0\\1\\0\\0\end{bmatrix}, \quad \begin{bmatrix}0\\-5\\0\\5\\0\end{bmatrix}$$

Then the dimension of U is

A. 2

B. 3

C. 4

D. 5

E. 6

14. Let p_1, p_2, p_3 be the polynomials in P_2 defined by

$$p_1(x) = -2 - 2x + 2x^2, \;\; p_2(x) = -3 + 7x - 17x^2, \;\; p_3(x) = 3 - 3x + (6+k)x^2.$$

Then p_1, p_2, p_3 are linearly independent if and only if k is different from

A. -3

B. 3

C. -2

D. 2

E. 0

15. Consider the following sets:

(i) Polynomials of the form $ax^2 + 1$ in P_2, where a varies in $\mathbb{R}$.
(ii) Points of the form (x, y) with $y \neq 0$ in $\mathbb{R}^2$.
(iii) Solutions of $y^{(3)} - xy'' + 5y = 0$ in $C^3([1, \infty))$.
(iv) Vectors (x, y, z) in $\mathbb{R}^3$ such that $5x = 2y - z$.

Among those sets, which ones are subspaces

A. (i) and (iii)

B. (ii), (iii) and (iv)

C. (i), (iii) and (iv)

D. (i) and (iv)

E. (iii) and (iv)

16. If a linear transformation $T : V \to V$ satisfies

$$T(2\mathbf{v}_1 + 3\mathbf{v}_2) = \mathbf{v}_1 + \mathbf{v}_2,$$
$$T(2\mathbf{v}_1 + \mathbf{v}_2) = 3\mathbf{v}_1 - \mathbf{v}_2,$$

then $T(-2\mathbf{v}_1 + \mathbf{v}_2)$ is

A. $3\mathbf{v}_1 + \mathbf{v}_2$

B. $-5\mathbf{v}_1 + 3\mathbf{v}_2$

C. $-2\mathbf{v}_1 + 2\mathbf{v}_2$

D. $\mathbf{v}_1 - 3\mathbf{v}_2$

E. $-2\mathbf{v}_1 + \mathbf{v}_2$

17. Let $A = \begin{bmatrix} -1 & 2 & 2 \\ -4 & 5 & 2 \\ -4 & 2 & 5 \end{bmatrix}$. Then,

A. A has eigenvector $(2, 5, 1)^T$

B. A has three linearly independent eigenvectors

C. $\lambda = 3$ has algebraic multiplicity two

D. $\lambda = -3$ is an eigenvalue of A

E. A is defective.

18. Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$ and let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by $T(\mathbf{v}) = A\mathbf{v}$. Then the difference $\dim(\mathrm{Rng}(T)) - \dim(\ker(T))$ is

A. -3

B. -1

C. 0

D. 1

E. 3

19. Consider the spring-mass system whose motion is governed by the initial value problem

$$y'' + 16y = 0 \qquad y(0) = 1, \quad y'(0) = 4\sqrt{3}.$$

Determine the position of the mass at time $t = \dfrac{\pi}{8}$.

A. $y = 0$

B. $y = -\sqrt{3}$

C. $y = \sqrt{3}$

D. $y = 1$

E. $y = -1$

20. The function $y_1 = t$ is a solution of the differential equation

$$t^2 y'' + 5ty' - 5y = 0, \qquad t > 0.$$

Choose a function y_2 from the list below so that the pair $\{y_1, y_2\}$ forms a fundamental set of solutions to the differential equation.

A. $y_2 = t^5$

B. $y_2 = t^4$

C. $y_2 = t^{-5}$

D. $y_2 = t^{-4}$

E. $y_2 = t^3$

21. Using the method of variation of parameter, a particular solution to

$$y'' + 16y = 4\sec(4t)$$

is $y_p(t) = u_1(t)\cos(4t) + u_2(t)\sin(4t)$. Then $u_2(t) =$

A. 1

B. t

C. $\ln|\sin 4t|$

D. $\ln|\cos 4t|$

E. $\sec(4t)$

22. If the method of undetermined coefficients is to be used, the suitable form for a particular solution $y_p(t)$ of the differential equation

$$y^{(4)} - y = e^{-t} + 3\sin(t)$$

is

A. $y_p(t) = Ate^{-t} + B\cos(t) + C\sin(t)$

B. $y_p(t) = At^2e^{-t} + B\cos(t) + C\sin(t)$

C. $y_p(t) = Ate^{-t} + Bt\cos(t) + Ct\sin(t)$

D. $y_p(t) = At^2e^{-t} + Bt\cos(t) + Ct\sin(t)$

E. $y_p(t) = Ate^{-t} + Bt\sin(t)$

23. Let $x(t)$ and $y(t)$ be the solutions of the following initial value problem:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \quad \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

Then $y(1)$ is equal to:

A. $4e^2 - e^3$

B. $e^2 + e^3$

C. $-4e^2 + 2e^3$

D. $4e^2 + e^3$

E. $-4e^2 - 2e^3$

24. The real 2×2 matrix A has an eigenvalue $\lambda_1 = 2 + i$ with corresponding eigenvector $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2+i \end{bmatrix}$. Then the REAL general solution to the system $\mathbf{x}' = A\mathbf{x}$ is

A. $e^{2t}(c_1 \begin{bmatrix} \cos t \\ -2\cos t - \sin t \end{bmatrix} + c_2 \begin{bmatrix} \sin t \\ \cos t - 2\sin t \end{bmatrix})$

B. $e^{2t}(c_1 \begin{bmatrix} \cos t \\ \cos t - 2\sin t \end{bmatrix} + c_2 \begin{bmatrix} \sin t \\ 2\cos t + \sin t \end{bmatrix})$

C. $e^{2t}(c_1 \begin{bmatrix} \cos t \\ \cos t - \sin t \end{bmatrix} + c_2 \begin{bmatrix} \sin t \\ 5\cos t - \sin t \end{bmatrix})$

D. $e^{2t}(c_1 \begin{bmatrix} 3\cos t \\ 2\cos t - \sin t \end{bmatrix} + c_2 \begin{bmatrix} -2\sin t \\ \cos t + \sin t \end{bmatrix})$

E. $e^{2t}(c_1 \begin{bmatrix} -2\cos t \\ \cos t - \sin t \end{bmatrix} + c_2 \begin{bmatrix} 3\sin t \\ \cos t + \sin t \end{bmatrix})$

25. If a fundamental matrix for $\mathbf{x}' = A\mathbf{x}$ is $X(t) = \begin{bmatrix} e^{-2t} & 0 \\ e^{-2t} & e^{2t} \end{bmatrix}$, then the general solution to the system of ODEs $\mathbf{x}' = A\mathbf{x} + \begin{bmatrix} 3e^{t} \\ 0 \end{bmatrix}$ is

A. $\begin{bmatrix} e^{-2t} & 0 \\ e^{-2t} & e^{2t} \end{bmatrix} \left\{ \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} 3e^{3t} \\ 3e^{-t} \end{bmatrix} \right\}$;

B. $\begin{bmatrix} e^{-2t} & 0 \\ e^{-2t} & e^{2t} \end{bmatrix} \left\{ \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} 3e^{3t} \\ -3e^{-t} \end{bmatrix} \right\}$;

C. $\begin{bmatrix} e^{-2t} & 0 \\ e^{-2t} & e^{-2t} \end{bmatrix} \left\{ \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} e^{-t} \\ 3e^{3t} \end{bmatrix} \right\}$;

D. $\begin{bmatrix} e^{-2t} & 0 \\ e^{-2t} & e^{-2t} \end{bmatrix} \left\{ \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} e^{3t} \\ e^{-t} \end{bmatrix} \right\}$;

E. $\begin{bmatrix} e^{-2t} & 0 \\ e^{-2t} & e^{2t} \end{bmatrix} \left\{ \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} e^{3t} \\ 3e^{-t} \end{bmatrix} \right\}$.