MA271, Fall 2000
Midterm 1

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- This booklet has SIX QUESTIONS, each worth 10 points. You have 50 minutes to do this test. Plan your time well. Read the questions carefully.

- This test is closed books and closed notes.

- Any calculator is allowed. But it cannot be used for justifying answers.

- In order to get full credit, your answers need to be correct and simplified. You need to explain in a comprehensible way how you obtained them.

- Both sides of the paper may be used. Indicate which problem you work on whenever you use a backside.
Question 1.
Consider a cube with side length $a = 1$ and corners as indicated in the picture.
Find the area of the triangle with corners A, B and C.
Question 2.
The curves\[ y = x^2 - 1 \]
and\[ y = -x^2 + 1 \]
intersect in the points $(1, 0)$ and $(-1, 0)$. Find the angle between the curves in the point $(1, 0)$. 
Question 3.
Find \( \lim_{n \to \infty} n \cdot (1 - \cos \left( \frac{1}{n} \right)) \).
Question 4.
Determine the convergence/divergence of the following series and clearly indicate your reasoning.

(a) $\sum_{n=2}^{\infty} \frac{1}{n \cdot (\ln(n))^{1+\pi}}$ where $a < 0$.

(b) $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{n^3 \cdot 3^n}$.

(c) $\sum_{n=1}^{\infty} \left(\frac{n - 2}{n}\right)^{2n}$. 
Question 5.

(a) For the series \( \sum_{1}^{\infty} \frac{(x-s)^n}{\sqrt{n!}} \) find the radius of convergence and the region of convergence.

(b) Also, find the value of the series for \( x = 2 \) with an error smaller than \( \frac{1}{100} \).
Question 6.

(a) Find the radius of convergence of the power series \( f(x) = \sum_{i=1}^{\infty} \left( \frac{x^2 + 1}{9} \right)^n \).

(b) Express \( f \) as a function of \( x \) without using the sum notation.

(c) Find \( f'(2) \).