NAME Answer Keys

10-digit PUID # ____

RECITATION INSTRUCTOR ____

RECITATION TIME ____

LECTURER ____

INSTRUCTIONS

1. There are 10 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. Also write your name at the top of pages 2–10.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet. No partial credit will be given.
4. No books, notes, calculators, or any electronic devices may be used on this exam.
5. Each problem is worth 8 points. The maximum possible score is 200 points.
6. Using a #2 pencil, fill in each of the following items on your scantron sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION NUMBER, put 0 in the first column and then enter the 3-digit section number. For example, for section 016 write 0016, and fill in the little circles.
   (c) On the bottom, under TEST/QUIZ NUMBER, write 01 and fill in the little circles.
   (d) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your 10-digit PUID, and fill in the little circles.
   (e) Using a #2 pencil, put your answers to questions 1–25 on your scantron sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
7. After you have finished the exam, hand in your scantron sheet and your test booklet to your recitation instructor.

1. The domain of $f(x) = \sqrt{2e^x - e^{2x})}$ is

A. $x \le \ln 2$

B. $0 < x \le \ln 2$

C. $0 < x$

D. $1 < x < e$

E. $e < x$

2. $\lim_{x \to 0+} \left(5x \sin \frac{1}{x} + 6 \frac{\sin x}{x} \right) =$

A. 1

B. 0

C. 5

D. 6

E. 11

3. Find the values of a and b for which the function is continuous for all x.

$$f(x) = \begin{cases} 2, & \text{if } x < 0 \\ ax + b, & \text{if } 0 \le x \le 3 \\ 1, & \text{if } 3 < x \end{cases}$$

A. $a = 3, \quad b = 1$

B. $a = 2, \quad b = 3$

C. $a = 1, \quad b = \frac{1}{2}$

D. $a = 2, \quad b = \frac{1}{6}$

E. $a = -\frac{1}{3}, \quad b = 2$

4. $\lim_{x \to 0} \dfrac{\sqrt{3x^2+16}-4}{x^2} =$

A. 0

B. $\dfrac{3}{8}$

C. $\sqrt{3}$

D. 4

E. Does not exist.

5. The equation $x^4 - x - 4 = 0$ has a root in the interval

A. $(-1, 0)$

B. $(0, 1)$

C. $(1, 2)$

D. $(2, 3)$

E. $(-\infty, -2)$

6. Find an equation of the tangent line to the curve $x^2 + xy + y^2 = 3$ at the point $(x, y) = (1, 1)$.

A. $y = 2x + 2$

B. $y = -x - 1$

C. $y = 2x - 1$

D. $y = -x + 2$

E. $y = x - 1$

7. If $f(x) = x(\ln x)^2$, then $f''(e) =$

A. $\frac{1}{e}$

B. $\frac{4}{e}$

C. $2 + e$

D. $3e^2$

E. $e + \frac{2}{e}$

8. If $f(x) = \frac{2x-1}{4x+2}$, find the inverse function $f^{-1}(x)$.

A. $f^{-1}(x) = \frac{2-x}{1-4x}$

B. $f^{-1}(x) = \frac{4x+2}{2x-1}$

C. $f^{-1}(x) = \frac{x-1}{4x-1}$

D. $f^{-1}(x) = \frac{2x+1}{2-4x}$

E. $f^{-1}(x) = \frac{x+1}{4x-2}$

9. If $f(x) = \sin x + \cos x$, then $\lim_{x \to \frac{\pi}{2}} \frac{f(x) - f(\frac{\pi}{2})}{x - \frac{\pi}{2}} =$

A. 0

B. -1

C. 1

D. $\frac{\pi}{2}$

E. Does not exist.

10. A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 2 ft/sec, how fast is the angle between the top of the ladder and the wall changing when the angle is $\frac{\pi}{4}$ rad?

A. $\frac{\sqrt{2}}{5}$ rad/sec

B. $\frac{2}{5}$ rad/sec

C. $\frac{2\sqrt{2}}{5}$ rad/sec

D. $\frac{\sqrt{3}}{15}$ rad/sec

E. $\frac{2\sqrt{3}}{5}$ rad/sec

11. If we use the linear approximation for $f(x) = \sqrt{x}$ at $a = 25$, then the estimate for the number $\sqrt{24.8}$ is

A. 4.94

B. 4.96

C. 4.98

D. 4.99

E. 5.00

12. The sum of two positive numbers is 16. What is the smallest possible value of the sum of their squares?

A. 50

B. 72

C. 98

D. 128

E. 162

13. The derivative of the function f is given by

$$f'(x) = (x-1)x(x+1)(x+2)$$

The function f has a local maximum only at

A. $x = -2$

B. $x = 0$

C. $x = -1$ and $x = 1$

D. $x = -1$ and $x = 0$

E. $x = -2$ and $x = 0$

14. The second derivative of the function f is given by

$$f''(x) = (x+3)(x+1)^2 x^3 (x-1)^4 (x-3)^5$$

How many inflection points does the graph of $y = f(x)$ have?

A. 0

B. 1

C. 2

D. 3

E. 4

15. On the interval $(0,4)$ the function $f(x) = 2x^3 - 9x^2 + 12x + 10$ is

A. increasing on $(0,1)$, and decreasing on $(1,4)$

B. decreasing on $(0,1)$, and increasing on $(1,4)$

C. decreasing on $(0,1)$, increasing on $(1,2)$, and decreasing on $(2,4)$

D. increasing on $(0,1)$, decreasing on $(1,2)$, and increasing on $(2,4)$

E. decreasing on $(0,1)$, increasing on $(1,3)$, and decreasing on $(3,4)$

16. $\lim_{x\to\infty} (e^{2x} + x)^{\frac{1}{x}} =$

A. 1

B. e^2

C. e

D. 2

E. ∞

17. A particle moves in a straight line and has velocity given by $v(t) = 3\sqrt{t+1} - \dfrac{1}{\sqrt{t+1}}$ and initial position $s(0) = 1$. Find its position function $s(t)$.

A. $s(t) = 2(t+1)^{3/2} - 2(t+1)^{1/2} + 1$

B. $s(t) = (t+1)^{3/2} - 2(t+1)^{1/2} + 2$

C. $s(t) = 2(t+1)^{3/2} + (t+1)^{1/2} - 2$

D. $s(t) = 2(t+1)^{3/2} - 2(t+1)^{1/2}$

E. $s(t) = 2(t+1)^{3/2} - (t+1)^{1/2}$

18. If m and n are constants, $\lim_{x\to 0} \dfrac{\cos(mx) - \cos(nx)}{x^2} =$

A. $m - n$

B. $n - m$

C. $m^2 - n^2$

D. $\dfrac{n^2 - m^2}{2}$

E. ∞

19. $\dfrac{d}{dx}\displaystyle\int_0^{x^3} e^{t^2}\, dt =$

A. $x^3 e^{x^6}$

B. $3x^2 e^{x^6}$

C. $3x e^{x^2}$

D. e^{x^6}

E. $3e^{x^2}$

20. $\displaystyle\int_0^{\frac{\pi}{6}} \tan^5 x \sec^2 x\, dx =$

A. $\dfrac{1}{27}$

B. $\dfrac{1}{54}$

C. $\dfrac{1}{45}$

D. $\dfrac{1}{162}$

E. $\dfrac{1}{324}$

21. Find the area of the region between the graph of $y = \dfrac{1}{x \ln x}$ and the x–axis from $x = e$ to $x = e^3$.

A. $3e - 1$

B. $\ln 3 + 1$

C. $\ln(3 + e)$

D. $\ln(e^3 - e)$

E. $\ln 3$

22. If $g(x) = \int_{x^2}^{x^4} \ln t \, dt$, then $g'(x) =$

A. $(16x^3 - 4x) \ln x$

B. $\dfrac{1}{x^4} - \dfrac{1}{x^2}$

C. $(16x^4 - 4x^2) \ln x$

D. $\ln x^4 - \ln x^2$

E. $\dfrac{\ln x^4 - \ln x^2}{8}$

23. A year ago there were 9 grams of a radioactive substance. Today there are 3 grams. The half–life of the substance in years is

A. $\dfrac{\ln 3}{\ln 9}$

B. $\dfrac{\ln 2}{\ln 3}$

C. $\dfrac{\ln 3}{\ln 4}$

D. $\ln 3$

E. $\ln 9$

24. The parabola $x^2 + 4x - 8y + 28 = 0$ has focus at

A. $(-2, 6)$
B. $(-1, 5)$
C. $(-1, 4)$
D. $(-2, 4)$
E. $(-2, 5)$

25. The ellipse $9x^2 + 4y^2 - 36x + 8y + 4 = 0$ has vertices at the points

A. $(2, -\sqrt{5})$ and $(2, \sqrt{3})$
B. $(-2, -4)$ and $(-2, 2)$
C. $(-2, 1)$ and $(-2, 6)$
D. $(2, -4)$ and $(2, 2)$
E. $(-4, 2)$ and $(2, 2)$