\[ \sin(u + v) = \sin u \cos v + \cos u \sin v \]
\[ \cos(u + v) = \cos u \cos v - \sin u \sin v \]
\[ \tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} \]
\[ \sin(2u) = 2\sin u \cos u \]
\[ \cos(2u) = \cos^2 u - \sin^2 u \]
\[ \sin^2 \theta + \cos^2 \theta = 1 \]
\[ 1 + \tan^2 \theta = \sec^2 \theta \]
\[ 1 + \cot^2 \theta = \csc^2 \theta \]

\[ \sin(u - v) = \sin u \cos v - \cos u \sin v \]
\[ \cos(u - v) = \cos u \cos v + \sin u \sin v \]
\[ \tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v} \]
1. Given $\triangle ABC$ with $\gamma = 90^\circ, \alpha = 30^\circ$, and $b = 15$, find the exact value of side $a$.

   A. $15\sqrt{3}$
   
   B. $10\sqrt{3}$
   
   C. $\frac{5}{\sqrt{3}}$
   
   D. $\frac{15}{2}$
   
   E. $5\sqrt{3}$

2. Given $\triangle ABC$ with $\gamma = 90^\circ, \text{angle} \alpha = 21^\circ$, and side $b = 8.6$, approximate the perimeter of the triangle to the nearest tenth.

   A. 17.4
   
   B. 21.1
   
   C. 19.9
   
   D. 18.1
   
   E. 23.6

3. Given the indicated parts of $\triangle ABC$ with $\gamma = 90^\circ$, express the third part in terms of the first two.

   $a, \beta, c$

   A. $c = a \tan \beta$
   
   B. $c = a \csc \beta$
   
   C. $c = a \sec \beta$
   
   D. $c = a \cos \beta$
   
   E. $c = a \sin \beta$
4. From a distance of 1 mile on level ground, a certain tower has an angle of elevation of 8°. Determine its height to the nearest foot. (1 mile = 5280 feet)

A. 742 feet
B. 836 feet
C. 734 feet
D. 826 feet
E. None of the above

\[
\tan 8^\circ = \frac{h}{5280} \\
h = 5280 \tan 8^\circ \\
h = 742.0556
\]

5. A ladder, 30 feet long, leans against the side of a building, and the angle between the ladder and the building is 16°.

If the distance from the bottom of the ladder to the building is increased by 3.0 feet, approximate the angle the ladder now makes with the building to the nearest 0.1°.

A. 10.1°
B. 10.5°
C. 22.1°
D. 21.7°
E. None of the above

\[
\sin 16^\circ = \frac{x}{30} \\
x = 30 \sin 16^\circ \\
x = 8.2691
\]

\[
x + 3 = 11.2681 \\
\sin \theta = \frac{11.2681}{30} \\
\theta = \sin^{-1} \left( \frac{11.2681}{30} \right) \\
\theta = 22.0637^\circ
\]
Questions 6 and 7. The following is a love story of two ships that do not pass in the night.

A ship leaves port at 2:00 pm and sails in the direction N 41° E at a rate of 37 mph. Another ship leaves the same port at the same time and sails in the direction N 49° W at a rate of 20 mph.

\[
\begin{align*}
D &= rt, \quad t = 4 \text{ hrs} \\
D_1 &= 37(4) = 148 \text{ mi} \\
D_2 &= 20(4) = 80 \text{ mi}
\end{align*}
\]

6. To the nearest whole mile, approximately how far apart are the ships at 6:00 pm?

A. 136 miles  
B. 126 miles  
C. 175 miles  
D. 168 miles  
E. None of the above

\[
x^2 = 148^2 + 80^2 \\
x = 168.2 \text{ mi}
\]

7. What is the bearing, to the nearest degree, from the first ship to the second?

A. S78°W  
B. S66°W  
C. S69°W  
D. S13°W  
E. None of the above

\[
\begin{align*}
\tan \alpha &= \frac{80}{148} \\
\alpha &= \tan^{-1}\left(\frac{80}{148}\right) \\
\alpha &= 28.393^\circ \\
\theta &= 111.0^\circ \quad 69.3930^\circ
\end{align*}
\]
8. Find all solutions of the equation using \( n \) as an arbitrary integer.

\[
\tan \theta = -\sqrt{3}
\]

A. \( \theta = \frac{2\pi}{3} + \pi n \)

B. \( \theta = \frac{\pi}{3} + 2\pi n \)

C. \( \theta = \frac{2\pi}{3} + 2\pi n \)

D. \( \theta = \frac{\pi}{3} + \pi n \)

E. None of the above

9. Find all solutions of the equation using \( n \) as an arbitrary integer.

\[
\sin(2x) = \frac{\sqrt{3}}{2}
\]

A. \( x = \frac{\pi}{6} + \pi n, \frac{5\pi}{6} + \pi n \)

B. \( x = \frac{\pi}{3} + \pi n \)

C. \( x = \frac{\pi}{3} + \pi n, \frac{5\pi}{6} + \pi n \)

D. \( x = \frac{\pi}{6} + \pi n, \frac{5\pi}{6} + \pi n \)

E. None of the above
10. Find all solutions of the equation in the interval \([0, 2\pi)\)

\[
\cos\left(2x - \frac{\pi}{4}\right) = 0
\]

A. \(\theta = \frac{5\pi}{24}, \frac{17\pi}{24}, \frac{29\pi}{24}, \frac{41\pi}{24}\)

B. \(\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}\)

C. \(\theta = \frac{7\pi}{24}, \frac{19\pi}{24}, \frac{31\pi}{24}, \frac{43\pi}{24}\)

D. \(\theta = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}\)

E. None of the above

\[
2x - \frac{\pi}{4} = \frac{\pi}{2} + n\pi
\]

\[
2x = \frac{3\pi}{4} + n\pi
\]

\[
X = \frac{3\pi}{8} + \frac{n\pi}{2}
\]

\[
X = \frac{3\pi}{8} + \frac{4\pi}{8}
\]

\[
X = \frac{3\pi}{8} + \frac{4\pi}{8}
\]

\[
X = \frac{3\pi}{8} + \frac{4\pi}{8}
\]

11. Find all solutions of the equation in the interval \([0, 2\pi)\)

\[
2\sin^2 \theta - 3\sin \theta + 1 = 0
\]

A. \(\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\)

B. \(\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}\)

C. \(\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}\)

D. \(\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}\)

E. None of the above

\[
(2\sin \theta - 1)(\sin \theta - 1) = 0
\]

\[
2\sin \theta - 1 = 0
\]

\[
\sin \theta = \frac{1}{2}
\]

\[
\theta = \frac{\pi}{6}, \frac{5\pi}{6}
\]

\[
\theta = \frac{\pi}{12}, \frac{5\pi}{6}
\]
12. Express as a trigonometric function of one angle.

\[
\sin(35^\circ)\cos(15^\circ) - \cos(35^\circ)\sin(15^\circ)
\]

A. \(\cos(50^\circ)\)

B. \(\sin(20^\circ)\)

C. \(\cos(20^\circ)\)

D. \(\sin(50^\circ)\)

E. None of the above

\[
\sin(u-v)
\]

\[
\sin(35^\circ - 15^\circ)
\]

\[
\sin(20^\circ)
\]

13. Find all solutions of the equation in the interval \([0, 2\pi]\).

\[
\sin t - \sin 2t = 0
\]

A. \(t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}\)

B. \(t = 0, \pi, \frac{5\pi}{3}, \frac{\pi}{3}\)

C. \(t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}, \frac{\pi}{3}\)

D. \(t = 0, \pi, \frac{2\pi}{3}, \frac{4\pi}{3}\)

E. None of the above

\[
\sin t - 2\sin t + \cos t = 0
\]

\[
\sin t(1 - 2\cos t) = 0
\]

\[
\sin t = 0 \quad 1 - 2\cos t = 0
\]

\[
\cos t = \frac{1}{2}
\]

\[
t = \frac{\pi}{3}, \frac{5\pi}{3}
\]

\[
t = \frac{7\pi}{3}, \frac{5\pi}{3}
\]
14. If \( \sin \alpha = -\frac{4}{5} \) and \( \cos \beta = \frac{7}{25} \), for a third-quadrant angle \( \alpha \) and a first-quadrant angle \( \beta \), find \( \cos(\alpha + \beta) \).

\[
\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta
\]

A. \( -\frac{4}{5} \)

B. \( -\frac{117}{125} \)

C. \( \frac{44}{125} \)

D. \( \frac{3}{5} \)

E. None of the above

15. Find the exact value of \( \cos 2\theta \) if \( \tan \theta = -\frac{5}{6} \) and \( 270^\circ < \theta < 360^\circ \)

\[
\cos 2\theta = \cos^2 \theta - \sin^2 \theta
\]

A. \( -\frac{11}{61} \)

B. \( -\frac{60}{61} \)

C. \( \frac{5}{61} \)

D. \( \frac{60}{61} \)

E. None of the above
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<td>3.</td>
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<td>C</td>
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<td>4.</td>
<td>742 feet</td>
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<td>5.</td>
<td>$22.1^\circ$</td>
<td>C</td>
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<td>6.</td>
<td>168 miles</td>
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<td>7.</td>
<td>$S69^\circ W$</td>
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<td>8.</td>
<td>$\theta = \frac{2\pi}{3} + \pi n$</td>
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