MA 15400
Fall 2013
Exam 3

LAW OF SINES:
\[
\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}
\]

Law of Cosines:
\[
\begin{align*}
a^2 &= b^2 + c^2 - 2bc \cos \alpha \\
b^2 &= a^2 + c^2 - 2ac \cos \beta \\
c^2 &= a^2 + b^2 - 2ab \cos \gamma
\end{align*}
\]

Double Angle Formulas:
\[
\begin{align*}
\sin(2u) &= 2 \sin u \cos u \\
\cos(2u) &= \cos^2 u - \sin^2 u \\
\tan(2u) &= \frac{2 \tan u}{1 - \tan^2 u}
\end{align*}
\]

Quadratic Formula:
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Angle Between Two Vectors:
\[
\cos \theta = \frac{\langle \vec{a} \rangle \cdot \langle \vec{b} \rangle}{|\vec{a}| |\vec{b}|}
\]
1. Find the exact value of the expression.

\[ \tan^{-1} \left( \tan \left( \frac{4\pi}{3} \right) \right) = \tan^{-1} \left( \sqrt{3} \right) = \frac{\pi}{3} \]

A. \( -\frac{\pi}{3} \)
B. \( \frac{4\pi}{3} \)
C. \( \frac{\pi}{3} \)
D. \( \frac{5\pi}{3} \)
E. None of the above

2. Write the expression as an algebraic expression in \( x \) for \( x > 0 \).

\[ \sin \left( 2\cos^{-1} x \right) = \frac{2 \sin x \cos x}{1 - x^2} = 2 \sin x \cos x \]

A. \( 2x - 2x^2 \)
B. \( 1 - x^2 \)
C. \( -1 + 2x \)
D. \( 2\sqrt{1-x^2} \)
E. None of the above

3. Use inverse trigonometric functions to find the solutions of the equation that are in the given interval, and approximate the solutions to four decimal places.

\[ 3\cos^2 x + 4\cos x - 5 = 0 \quad [0, 2\pi) \]

A. \( 2.4756, 3.8076 \)
B. \( 0.6660, 5.6172 \)
C. \( 0.9048, 2.2368 \)
D. \( 4.0464, 5.3784 \)
E. None of the above
This would be a good time to check the mode on your calculator!

4. Given \( \triangle ABC \) with \( \alpha = 90^\circ, \gamma = 25^\circ, \) and \( a = 21.5 \). Find the values of sides \( b \) and \( c \) rounded to one decimal place.

   A. \( b = 18.4 \) and \( c = 9.2 \)
   B. \( b = 18.9 \) and \( c = 8.1 \)
   C. \( b = 18.9 \) and \( c = 9.2 \)
   D. \( b = 18.4 \) and \( c = 8.1 \)
   E. None of the above

5. A surveyor notes that the direction from point A to point B is N50°W and the direction from A to point C is S30°W. The distance from A to B is 275 yards, and the distance from B to C is 450 yards. Approximate the distance from A to C to the nearest yard.

   A. 324 yards
   B. 297 yards
   C. 312 yards
   D. 289 yards
   E. None of the above

6. An airplane flies 333 miles from point A in the direction 50° and then travels in the direction 200° for 222 miles. To the nearest mile, approximately how far is the airplane from A?

   A. 267 miles
   B. 198 miles
   C. 248 miles
   D. 179 miles
   E. None of the above

\[
x^2 = 333^2 + 222^2 - 2(333)(222) \cos 30^\circ
\]
\[
x = \sqrt{179.2468} m.
\]
7. Given $\triangle ABC$ with $\alpha = 51^\circ$, $a = 375$ and $c = 452$, which statement is true?
   A. There exist two possible triangles and one of the values of $b = 153.2$.
   B. There only exists one possible triangle and $\beta = 59.5^\circ$.
   C. There exist two possible triangles and one of the values of $\gamma = 112.3^\circ$.
   D. There only exists one possible triangle and $b = 421.7$.
   E. There is not enough information to solve for the rest of the triangle.

8. Given $a = (4,6)$ and $b = (-5,2)$ find $3a - 4b$.
   A. $(-31,-18)$
   B. $(-8,36)$
   C. $(32,10)$
   D. $(8,-36)$
   E. None of the above

9. Given vector $c = -8i + 4j$ find $|c|$ to the nearest tenth.
   A. $|c| = 8.7$
   B. $|c| = 8.1$
   C. $|c| = 8.3$
   D. $|c| = 8.5$
   E. None of the above
10. The magnitudes and directions of two forces acting at a point P are...

\[ 4.4 \text{ lb}, \ 50^\circ \]
\[ 9.7 \text{ lb}, \ 165^\circ \]

Approximate the direction of the result vector to the nearest whole degree.

A. \( \theta = 129^\circ \)
B. \( \theta = 132^\circ \)
C. \( \theta = 134^\circ \)
D. \( \theta = 138^\circ \)
E. None of the above

11. Find a vector that has the same direction as \((-8, 8)\) and five times its magnitude.

A. \((30, -40)\)
B. \((3, -4)\)
C. \((-30, 40)\)
D. \((-3, -4)\)
E. None of the above

12. Find the angle between the two vectors \(a = (-7, -6), \ b = (-2, 10)\).

Round to the nearest tenth of a degree.

A. 121.5°
B. 119.3°
C. 122.6°
D. 113.9°
E. None of the above
13. Determine $m$ such that the two vectors $c = (2m, -4), d = (5, 6)$ are orthogonal.
   A. $m = 3.75$
   B. $m = -2.4$
   C. $m = -3.75$
   D. $m = 2.4$
   E. None of the above

   $\mathbf{c} \cdot \mathbf{d} = (2m)(5) + (-4)(6) = 10m - 24$
   $10m - 24 = 0$
   $10m = 24$
   $m = 2.4$

Questions 14 and 15. An airplane is flying in the direction $165^\circ$ with airspeed of 450 mph, and a 50 mph wind is blowing directly from the west.

14. Approximate the Ground Speed of the airplane to the nearest mph.
   A. 465 mph
   B. 441 mph
   C. 447 mph
   D. 471 mph
   E. None of the above

   $r = \sqrt{450^2 + 50^2} = \sqrt{(450)^2 + (50)^2}$
   $= \sqrt{202500 + 2500}$
   $= \sqrt{205000}$
   $\approx 453.4$ mph

15. Approximate the True Course of the airplane to the nearest whole degree.
   A. $157^\circ$
   B. $159^\circ$
   C. $151^\circ$
   D. $153^\circ$
   E. None of the above

   $\sin \alpha \approx 0.9339$
   $\alpha \approx 69.0^\circ$
   $\beta = \tan^{-1}(-0.38) + 180^\circ$
   $\beta \approx -21.0 + 180^\circ$
   $\beta \approx 159.0^\circ$
# Exam 3 Answers

1. C  \( \frac{\pi}{3} \)
2. D  \( 2\pi \sqrt{1 - x^2} \)
3. B  0.6660, 5.6172
4. A  \( b = 18.4 \) and \( c = 9.2 \)
5. C  312 yards
6. D  179 miles
7. A  There exist two possible triangles and one of the values of \( b = 153.2 \).  
8. C  (32,10)
9. E  \( |c| = 8.9 \)
10. D  \( \theta = 138^\circ \)
11. C  (\(-30,40\))
12. B  119.3\(^\circ\)
13. D  \( m = 2.4 \)
14. A  465 mph
15. B  159\(^\circ\)